

NEW CHARACTERIZATIONS OF SOME MEAN VALUES  
AND FUNCTIONAL EQUATIONS

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**Abstract.** New characterizations of a special function  $p$  for a symmetric mean  $M(a, b; p, r_{n,k}; q)$  and a certain functional equation are proved.

**MSC 2000.** 38B22.

**Key words.** Gauss' functional equation, monotonic function, power mean.

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