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LOEWNER CHAINS AND DIFFERENTIAL SUBORDINATIONS FOR FUNCTIONS OF ONE AND SEVERAL COMPLEX VARIABLES. QUASICONFORMAL EXTENSIONS AND APPLICATIONS IN FLUID MECHANICS

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Host institution of the project: Babeş-Bolyai University

A. Research team:

1. Professor Gabriela Kohr(director)
2. Professor Mirela Kohr
3. Associate professor Paula Curt
4. Assistant PhD Veronica Nechita
5. Assistant PhD Mădălina Bălăeţi (PhD student)

B. Project summary:

The thematic of this project is to approach up-to-date and modern problems in the theory of Loewner chains as well as of differential subordinations in the complex plane and in several complex variables, based on new results that have been obtained by the experienced members of this research team and by prestigious mathematicians in our country and abroad. Also, there is treated the problem of quasiconformal extension of quasiregular biholomorphic mappings which can be imbedded in Loewner chains. These problems are in a direct connection with recent researches in the geometric function theory of one and several complex variables that are treated in prestigious universities in the world. The purpose of this project is to create new specialists in the above domains. The approached domain is challenging and has practical implications in the study of certain problems in fluid mechanics.

Research directions:

- The study of biholomorphic mappings which have generalized parametric representation: geometric characterizations, growth and covering results, coefficient bounds.

- Quasiconformal extension for quasiregular biholomorphic mappings which can be imbedded in subordination chains.
- The application of the theory of subordination chains to introduce certain new subclasses of biholomorphic mappings on the unit ball in the n-dimensional complex space.
- The application of the theory of differential subordinations in the complex plane to introduce new differential and integral operators that preserve interesting geometric properties, as well as to obtain new global injectivity conditions.
- New subclasses of univalent functions on the unit disc or on different domains in the complex plane
- The thoroughgoing study of differential subordinations in $\mathbb{C}^n$.
- The application of the theory of univalent functions as well as of other modern methods to study certain problems related to incompressible fluid flows.

C. Objectives

2007

1. Differential subordinations in the complex plane. Applications
2. Geometrical characterizations of biholomorphic mappings that have parametric representation

Degree of achievement: 100%

2008

1. Quasiconformal extension of quasiregular biholomorphic mappings on the unit ball in $\mathbb{C}^n$.
2. Differential superordinations in the complex plane. Applications
3. New subclasses of univalent functions and the preservation of some analytical and geometrical properties, by using the theory of differential subordinations in the complex plane

Degree of achievement: 100%

2009

1. Generalizations of the Roper-Suffridge extension operator
2. Differential and integral operators that preserve geometrical and analytical properties
3. Global injectivity criteria for holomorphic mappings of one and several complex variables

Degree of achievement: 100%

2010

1. Subordination chains for mappings of one and several complex variables
2. Viscous incompressible fluid flow problems on simply connected domains in the complex plane, by using the theory of univalent functions
3. Special problems in the geometric function theory: extreme points and support points for families of biholomorphic mappings on the unit ball in $\mathbb{C}^n$

Degree of achievement: 100%

Table: Objectives and activities

D. Contributions of the PhD students

• Assistant (PhD student) **Mădălina Bălăeţi**, Department of Mathematics and Computer Science, Petroşani University

- M. Bălăeţi finished her PhD studies during the period of this grant, and in June 11 2010 she had the public presentation of her PhD thesis *Univalent functions of one and several complex variables*. Adviser Professor G. Sălăgean, Babeş-Bolyai University. She obtained the PhD degree in mathematics on July 28 2010. She has collaborated with the director of the grant to elaborate the chapters three and four of her PhD thesis related to geometric function theory of several complex variables. This PhD thesis deals with problems in a direct connection with the thematic area of the grant.

Research publications:

- **C.M. Bălăeţi**, *Differential superordinations defined by an integral operator*, J. Math. Appl., **31**(2009), 31-38 (BDI).
- **C.M. Bălăeţi**, *A special differential superordination in the complex plane*, Studia Univ. Babeş-Bolyai (Mathematica), 55(2010), 31-40 (BDI).
- **C. M. Bălăeţi**, *A general class of holomorphic functions defined by integral operator*, General Mathematics, 18, 2(2010), 59-69 (BDI).
- **C.M. Bălăeţi**, **V. Nechita**, *Loewner chains and almost starlike mappings of complex order* λ, Carpathian J. Math., 26(2010), 146-157 (ISI).
- **C. M. Bălăeţi**, *Almost starlikeness of complex order* λ *associated with extension operators for biholomorphic mappings*, Mathematica (Cluj), to appear (BDI).
- **C. M. Bălăeţi, V. Nechita**, *Applications of the Roper-Suffridge extension operator to almost starlike mappings of complex order* λ, submitted in 2010 to Complex Variables and Elliptic Equations (BDI).

In the PhD thesis and in the above papers there are investigated the following main subjects, in connection with the thematic area of this grant:

- New subclasses of univalent functions in the complex plane (in connection with the objectives 1/2007 and 2/2009)
- New applications of the theory of differential subordinations and superordinations in the complex plane (in connection with the objectives 1/2007 and 2/2008)
- New families of biholomorphic mappings on the unit ball in $\mathbb{C}^n$ by using the theory of Loewner chains. Analytical and geometrical characterizations of these families of biholomorphic mappings (in connection with the objective 1/2010)
- Applications of the generalized Roper-Suffridge extension operators to preserve certain univalence conditions from the one-dimensional case to several complex variables (in connection with the objective 2/2009)

Research visits

- Free University of Berlin, Department of Mathematics; invited by Prof. H. Begehr (September 2008).

  Seminar talk: **M. Bălăeţi**, *Loewner chains and almost starlikenes of complex order* λ.
- Technical University of Rzeszow, Department of Mathematics; invited by Prof. S. Kanas (May 2008).

  Seminar talk: **M. Bălăeţi**, *About differential superordinations defined by an integral operator.*

The main results have been communicated to research seminars of Department of Function Theory, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, and to the following conference:

- The 5th International Conference Dynamical Systems and Applications, 15-18 June, 2009, Constanţa.
  C.M. Bălăeţi, *An Integral Operator Associated with Differential Superordinations.*

E. Achievements

E1. ISI Publications

1. P. Duren, I. Graham, H. Hamada, **G. Kohr**, *Solutions for the generalized Loewner differential equation in several complex variables*, Math. Annalen, **347** (2010), 411–435 (ISI). Impact factor/2008: 1.027
2. H. Hamada, **G. Kohr**, *On some classes of bounded univalent mappings in several complex variables*, Manuscripta Mathematica, 131(2010), 487–502 (ISI). Impact factor/2008: 0.509; MR2592092
3. **P. Curt**, **G. Kohr**, **M. Kohr**, *Homeomorphic extension of strongly spirallike mappings in* $\mathbb{C}^n$, Science in China Ser. A Mathematics, 53(2010), no.1, 87-100 (ISI). Impact factor/2008: 0.408; MR2594749

4. H. Hamada, **G. Kohr**, P.T. Mocanu, I. Şerb, *Convex subordination chains and injective mappings in* $\mathbb{C}^n$, J. Math. Anal. Appl., **364** (2010), 32-40 (ISI). Impact factor/2008: 1.046; MR2576049

5. C.H. Chu, H. Hamada, T. Honda, **G. Kohr**, *Distortion theorems for convex mappings on homogeneous balls*, J. Math. Anal. Appl., 369 (2010), 437-442; DOI: 10.1016/j.jmaa.2010.03.014 (ISI). Impact factor/2008: 1.046.

6. H. Hamada, T. Honda, **G. Kohr**, *Bohr's theorem for holomorphic mappings with values in homogeneous balls*, Israel Journal of Mathematics, 173(2009), 177-187 (ISI). Impact factor/2008: 0.625; MR2570664

7. I. Graham, H. Hamada, **G. Kohr**, **M. Kohr**, *Spirallike mappings and univalent subordination chains in* $\mathbb{C}^n$, Ann. Scuola Norm. Sup. Pisa Classe di Scienze, Serie V, Vol. 7(2008), 717-740 (ISI). Impact factor/2008: 0.519; MR2483641 (2009m:32026)

8. **P. Curt**, **G. Kohr**, *Some remarks concerning quasiconformal extensions in several complex variables*, Journal of Inequalities and Applications Volume 2008, Article ID 690932, 16 pages (ISI). Impact factor/2008: 0.764; MR2415409 (2009d:32015)

9. I. Graham, H. Hamada, **G. Kohr**, *On subordination chains with normalization given by a time-dependent linear operator*, Complex Analysis and Operator Theory, DOI 10.1007/s11785-010-0106-1/2010, to appear.

10. **C.M. Bălăeţi**, **V. Nechita**, *Loewner chains and almost starlike mappings of complex order* λ, Carpathian J. Math., 26(2010), 146-157 (ISI).

E2. BDI Publications

1. I. Graham, H. Hamada, **G. Kohr**, *On non-normalized subordination chains in* $\mathbb{C}^n$, Mathematica (Cluj), 52(75)(2010), 153-164 (BDI).

2. **M. Kohr**, C. Pintea, W. Wendland, *On mapping properties of layer potential operators for Brinkman equations on Lipschitz domains in Riemaniann manifolds*, Mathematica (Cluj), 52(75), no. 1 (2010), 31-45 (BDI).

3. **C. M. Bălăeţi**, *A general class of holomorphic functions defined by integral operator*, General Mathematics, 18, 2(2010), 59-69 (BDI).

4. **C.M. Bălăeţi**, *A special differential superordination in the complex plane*, Studia Univ. Babeş-Bolyai (Mathematica), 55(2010), 31-40 (BDI).

5. **P. Curt**, *Loewner chains and quasiconformal extensions of holomorphic mappings in* $\mathbb{C}^n$, Mathematica (Cluj), 51(74)(2009), 143-151 (BDI).

6. **C.M. Bălăeţi**, *Differential superordinations defined by an integral operator*, J. Math. Appl., **31**(2009), 31-38.

7. **V. Nechita**, *On some classes of analytic functions defined by a multiplier transformation*, Studia Univ. Babeş-Bolyai, Mathematica, vol. 53, no. 3(2008), 69-74 (BDI).

8. **V. Nechita**, *Differential sandwich theorems for analytic functions defined by the Dziok-Srivastava linear operator*, Mathematica (Cluj), 50(73)(2008), 85-94 (BDI).

9. **P. Curt**, *Sufficient conditions for univalence and quasiconformal extensions in several complex variables*, Studia Univ. Babeş-Bolyai (Mathematica), vol. 55, nr. 4(2010), to appear (BDI).

10. **P. Curt**, D. Fericean, T. Groşan, *Φ-like functions in two-dimensional free boundary problems*, Mathematica (Cluj), to appear (BDI).

11. **V. Nechita**, *A univalence condition for analytic functions in the unit disc*, Mathematica (Cluj), to appear (BDI).

12. **C. M. Bălăeţi**, *Almost starlikeness of complex order λ associated with extension operators for biholomorphic mappings*, Mathematica (Cluj), to appear (BDI).

E3. Papers submitted for publication

1. P. Duren, H. Hamada, **G. Kohr**, *Two-point distortion theorems for harmonic and pluriharmonic mappings*, submitted.

2. L. Arosio, F. Bracci, H. Hamada and G. Kohr, *An abstract approach to Loewner's chains*, submitted.

3. I. Graham, H. Hamada, **G. Kohr**, **M. Kohr**, *Extreme points, support points and Loewner variation in several complex variables*, submitted.

4. **C. M. Bălăeţi, V. Nechita**, *Applications of the Roper-Suffridge extension operator to almost starlike mappings of complex order λ*, submitted in 2010 to Complex Variables and Elliptic Equations (BDI).

E4. Findings

- Report-2007
- Report-2008
- Report-2009
- Report-2010
- Presentation-2009

E5. Research visits

- University of Toronto, Department of Mathematics; invited by Prof. I. Graham (May 2008, August 2008, May 2009, May 2010); G. Kohr and M. Kohr.
- University of Stuttgart, Department of Applied Mathematics; invited by Prof. W.L. Wendland (March 2008; July and August 2009, July 2010); M. Kohr and G. Kohr.

- Free University of Berlin, Department of Mathematics; invited by Prof. H. Begehr (August 2008- P. Curt; September 2008-C.M. Bălăeţi; October 2008- G. Kohr; M. Kohr)

- Technical University of Rzeszow, Department of Mathematics; invited by Prof. S. Kanas (May 2008); C.M. Bălăeţi.

- University of Turku, Department of Mathematics; invited by Prof. M. Vuorinen (August 2009); P. Curt.

E6. Conferences (without financial support from this grant)

- NTHCA10 "New Trends in Harmonic and Complex Analysis", June 29-July 3, 2010, Bremen, Germany;
  G. Kohr, *Parametric representation and Loewner chains in several complex variables* (**invited talk**).

- Mini-courses in Mathematical Analysis 2010, Padova, June 21-25;
  M. Kohr, *Boundary value problems for Brinkman operators on Lipschitz domains. Applications* (communication).

- NTHCA10 New Trends in Harmonic and Complex Analysis, June 29 - July 3, 2010, Bremen, Germany;
  M. Kohr, *Boundary value problems for Brinkman operators on Lipschitz domains. Applications* (**invited talk**).

- Workshop on Interfaces in Multiphase Flow, Stuttgart, July 1st - July 2nd, 2010;
  M. Kohr, *Boundary value problems for Brinkman operators on Lipschitz domains - Applications* (**Invited speaker**).

- Modern Complex Analysis and Operator Theory and Applications, IV, El Escorial (Madrid), June 17-21, 2009;
  G. Kohr, *Loewner chains and the generalized Loewner differential equation on the unit ball in* $\mathbb{C}^n$ (**invited talk**).

- International Conference on Complex Analysis and Related Topics. The 12-th Romanian-Finnish Seminar, 17-21 August 2009, Turku (Finland);
  G. Kohr, *Solutions for the generalized Loewner differential equation and spirallike mappings in* $\mathbb{C}^n$ (**invited speaker**).

  M. Kohr, G.P. Raja Sekhar, W.L. Wendland, Boundary integral equations for two-dimensional low Reynolds number flow past a porous body.

- International Conference on Microfluidics and Complex Flows ECM 09, November 5-6, 2009, Tunis;
  M. Kohr, *Transmission problems for Stokes and Brinkman operators on arbitrary Lipschitz domains. Applications to porous media flow problems* (**invited speaker**).

- "Quantitative Models in Economics", May 15-16, 2009, Faculty of Economics and Business Admin., Babeş-Bolyai University, Cluj-Napoca;
  P. Curt, *A short introduction to stochastic Loewner evolutions.*

- The 5th International Conference Dynamical Systems and Applications, 15-18 Iunie, 2009, Constanţa;
  C. M. Bălăeţi, *An Integral Operator Associated with Differential Superordinations*

- 2008 One and Several Complex Variables Conference, Lexington (Kentucky), USA, 8-11 May 2008;
  G. Kohr, M. Kohr, *New results in the theory of univalent subordination chains in several complex variables. Geometric aspects.*

- Analysis, PDEs and Applications on the occasion of the 70th birthday of Vladimir Maz'ya, June 30 - July 4 2008, Rome;
  M. Kohr, *Boundary integral method for a Stokes flow past porous bodies.*

- International Conference on Complex Analysis and Related Topics. The XI-th Romanian-Finnish Seminar, Alba Iulia, Romania, August 14-19, 2008;
  G. Kohr, M. Kohr, *Subordination chains and generalized parametric representation in several complex variables* (**invited talk**).

- INDAM Workshop on Holomorphic Iteration, Semigroups, and Loewner Chains, Rome, 9–12 September 2008;
  G. Kohr, M. Kohr, *New aspects in the theory of Loewner chains in several complex variables* (**invited talk**).

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