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CREATIVITY AND INNOVATION IN EDUCATION-HEURISTIC PROCEDURES IN SOLVING PROBLEMS

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Abstract. As the title suggests, this paper is about some procedures that can be useful to mathematical problems solvers, and not only. Heuristics, although known in general, as a notion, is less applied in the usual routine. The article urges, through the proposed examples, to use heuristics in order to develop innovative thinking, regardless of the level of knowledge.

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1. INTRODUCTION

Heuristics bring together processes designed to lead to invention and discovery. School programs are mainly focused on the volume of information (although a formative education is desired), as a result, students have too few opportunities that require to a high degree both thinking and creative imagination, regulatory psychic processes in becoming their personality. The framework objectives we propose objectives to be fulfilled in this material are, mainly:

- O1) Acquiring some heuristic methods and procedures of wide applicability in solving problems with mathematical or practical content in various contexts.
- O2) Development of cognitive strategies (knowledge, application of heuristic procedure (h.p.), solutions based on the use of h.p., in solving problems with an increased degree of difficulty, atypical.
- O3) Extracting and formulating problems from previously analyzed problematic situations; the development of investigation-exploitation and resolution capacities in general.
- O4) Capitalizing on mathematical knowledge (in the form of cognitive schemes and operators) in solving problems that require heuristic solving strategies.
- O5) Strengthening the personality of the solver - by developing the attraction for the problem and discovering the logically hidden implications - the positive attitude towards the solving and the creative act of learning mathematics in general.

These framework objectives can be detailed in so-called specific or reference objectives, as follows:

Reference objectives (specific skills)

- O1.1. To explain the meaning of each h.p., e.m.(heuristic methods) and list them, accompanied by instructions for their application.
- O1.2. To classify h.p. depending on the moment, the stage during the solving act in which it is used, depending on the analytical/synthetic approach to the problem.
- O2.1. To identify the most convenient ones to be used in p.s.(problem solving) with practical and applied content.
- O2.2. To apply h.p. in finding r.i.(resolution idea), the resolution strategy.

Numerous works deal with ways of solving well-known or less well-known classic problems, more difficult or not. In the following we will present some examples of problems for different levels of preparation, for which we will apply one heuristic procedures.

2. SELECTED EXAMPLES

EXAMPLE 1. *Familiarization of students with heuristic procedures, namely: customization, analysis by synthesis and return to definitions*

The problem is proposed: *Let A and B be two movable points on a fixed circle C(O,R), so that AB=2a, a constant. What figure describes the middle of the segment (AB)?* Learning is supported, as shown in the table below:

- Teacher Strategy(IOG)
- Student strategy, conjectures issued, their justification
- Customization
- Analysis by synthesis
- Return to definitions, theorems
- Let us consider some particular positions of the segment (AB)
- Let's guess what figure M describes
- What can we deduce from the hypothesis?
- What rules would be useful here?

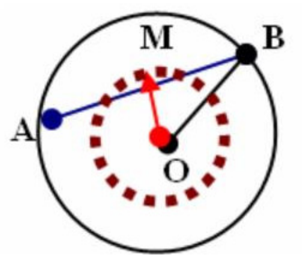

Fig. 2.1 – Trajectory of point M

The means of the segments are the vertices of a polygon (regular), trunk, square, hexagon, etc. From the right triangle BMO, the students intuit that $M \in C(O, R_1)$, with $R_1 = OM = \sqrt{R^2 - a^2}$, where: $a = MB = \frac{1}{2} \cdot AB$,...

Observations: Particularization, as a heuristic procedure, supports the students' intuition and inductive reasoning based on which the students have issued the conjecture that M travels a circle. In order to validate this conjecture through analytical thinking, another heuristic procedure is used: the analysis of a detail that is then integrated (synthesis) into the whole figure following the return to some definitions, useful theorems(e.g.).

Example 2. *Composing problems according to a previously elaborated strategy*

The problem is proposed:

(P) Let A, B, C $\in$ *(0,1) or A, B, C*>1 *and X, Y, Z*$\in$ *R, so* $Ax = BC$, *By* $= AC$, $Cz = AB$. *It is required to verify that* x+y+$z$$\geq$*6.* (County Stage, Sibiu, 1966) Solving has to select from below items:

(1) By reformulating the problem (P.E.) and IOG
(2) What can we deduce from the hypothesis?
(3) Let's reformulate the conclusion!

Students express: x $= log_A(BC), y = log_B(AC), z = log_C(AB)$, and writing equivalent, it is reached to a *problem previously solved*: $\sum Log_A(BC) \geq 6$.

It is proposed that the students compose a problem (as exercise) analogous to (P). They easily find the problem proposed at the Satu Mare, County Stage, 1993:

Let a,b,c,d $\in$*(0,1) or a,b,c,d* $\in$ *R, so that are verified:* $a^x = bcd, b^y = cda, c^z = adb, d^t = abc$. *It is required to show that:* $x+y+z+t \geq 12, \sum 1/(1+x) = 4$

Observations:

The heuristic process called "the reformulation of the problem" has led to a problem previously solved. The problematic space allows the generalization of the property (P). Generalization reveals the idea of solving and reduces (sometimes even eliminates) the heuristic character of the problem.

Example 3. *Registering the idea of solving, detecting the heuristic procedures used in solving, extracting the resolution strategy and composing problems on this strategy, by studying an already solved problem, using customization and generalization for this purpose.*

Mode of unfolding: Students are organized in groups, each group receives a problem solved for study-the problems differ-specifying, however, there are the same type of didactic tasks:

1) Explanation of the steps in the demonstration, the solution of the respective problem, of the calculations, completing the dotted spaces, etc.;

2) Discovery of I.R. and specification P.E. used by the author of the solution;

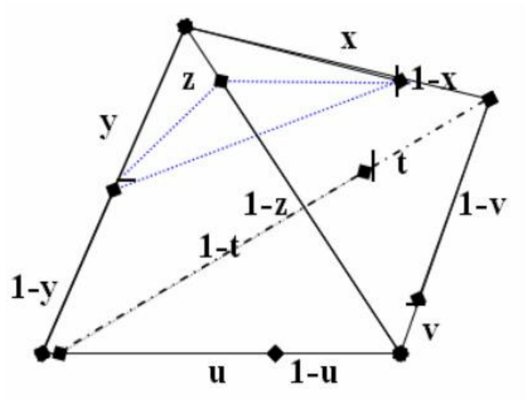

Fig. 2.2 – Resolution idea space representation

3) Extraction and concise exposure of the resolution strategy;

4) Composition of problems on the same idea of solving, resolution strategy.

For example:

The group that had the problem: $\forall x, y, z, t, u, v \in [0, 1]$ *verifies the inequality:* $x \cdot y \cdot z + u \cdot t \cdot (1 - x) + v \cdot (1 - z) \cdot (1 - t) \cdot (1 - u) \cdot (1 - v) \leq 1$

The group found I.R.: Left products represent the volumes u, v are the lengths of segments placed on the edges of the tetrahedron with one end in each tetrahedron, and the sum of the volumes of the four of them, does not exceed the volume of the given tetrahedron. Next, the students proposed inequalities in an equilateral and square triangle.

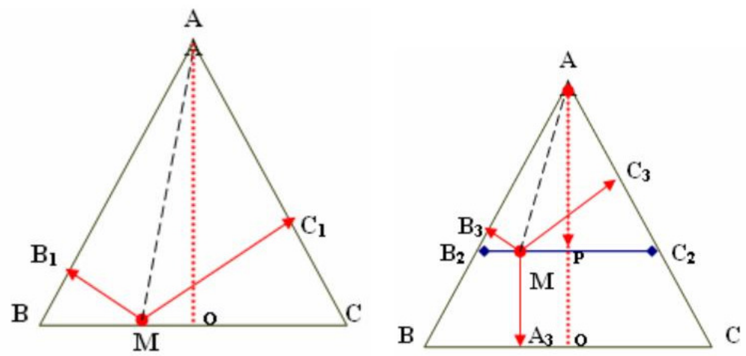

Fig. 2.3 – Applying customization(left) and generalization(right)

Among the problems considered by the students, it was selected(Schneider, 1996):

Let ABC an equilateral triangle. It is to demonstrate that, for any point M, placed in the interior of the triangle, the sum of the distances from M at the three sides is constant and equal to the height of the triangle.

1) Particularization of the problem(P.E.) Let's suppose that the point M does not belong to ABC but only to the side BC. The use of the property of the summative area is suggested to the students: $S_{AMB} + S_{AMC} = S_{ABC}$. Also, noting with B_1, C_1 the feet of the perpendicular from M, to the sides AB and AC of the triangle ABC, it results: $MB_1 + MC_1 = AO$, where: O is the foot of the height from A on BC.

2) Generalization. If M is inside the ABC triangle, we carry through M, a parallel to the side BC of the equilateral triangle. We note with B_2 and C_2 the intersections of this parallel with the sides AB, respectively AC, of the same triangle and with B_3, C_3, the distances of point M, on the same sides. The triangle AB_2C_2 is also equilateral (like the given one) and the areas property is similar: $S_{AMB_2} + S_{AMC_2} = S_{AB_2C_2}$.

We note with P the intersection of the B_2C_2 parallel with the height of A, the point identical to the foot of the height from A based on the equilateral triangle AB_2C_2. According to the previously demonstration, the area relation leads to: $MB_3 + MC_3 = AP$. MA_3 is the distance to the side BC, the difference to the length of the height from A, and completes the segment AO: $MA_3 + MB_3 + MC_3 = AP + PO = AO, \forall M \in int(ABC)$

It results: the sum of the distances from an interior point, to the sides of the equilateral triangle, is equal to the triangle height, therefore constant. Such inequalities, like previous, we are also thinking about to propose! It is relatively simple:

EXAMPLE 4. Assumption: x,y,z,t,u,v ∈ [0,1] Conclusion: x(1-t)+y(1-x)+z(1-y)+t(1-z)¡=1

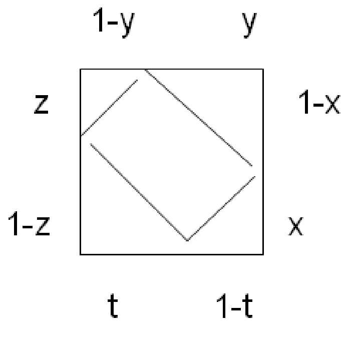

Fig. 2.4 – Image of graphical solving

The rectangle area is at least equal to $\frac{1}{2}$. The sum of the areas of the triangles results at most equal to $\frac{1}{2}$.

3. CONCLUSIONS

The authors of the article present "heuristic" as a discipline that brings together procedures meant to lead to invention and discovery. As the programs of the educational objects place the emphasis, mainly, on the volume of information (although the guidelines concern a formative education) students have too few occasions of discovery and rediscovery, of invention, of problematization, which require in a high degree both thinking and the creative imagination, especially the decisive regulatory psychic processes in becoming their personality. The article is addressed to those who love the atypical, non-standard problems, who like to discover non-solving ways in solving problems, who want to learn to think, ask questions in the heuristic approach to seek the solving idea. The volume of information necessary in solving the proposed problems does not exceed that of a high-level student of the 10th grade, including those from the pedagogical profile, who are preparing to teach others to think. The authors believe that the presented material can be enriched and adapted to be used by any specialized teacher.

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