

Volumes of Revolution

Determine the volume of the solid generated when the area bounded by the given curves is rotated about the stated axis.

1. $y = 4 - x^2$, x -axis, y -axis, quadrant I; about the x -axis.
2. $y = x^3$, $y = x$; about the y -axis.
3. $y = \sin x$, x -axis, $x = 0$, $x = \pi$; about the x -axis. Set up the integral, but do not evaluate.
4. $y = 2x - x^2$, $y = x^2$; about the x -axis. Set up the integral, but do not evaluate.
5. $y = x - 2$, $y = -\frac{1}{2}(x - 2)^2$; about x -axis. Set up the integral, but do not evaluate.
6. $y = \cos x$, $y = \sin x$, $x = 0$, $x = \frac{\pi}{2}$, about the x -axis. Set up the integral, but do not evaluate.
7. $y = \arcsin x$, $y = -\frac{\pi}{2}$, $y = \frac{\pi}{2}$, y -axis; about the y -axis. Set up the integral, but do not evaluate.
8. $y = x^2$, $x = y^2$ about the x -axis. Set up the integral, but do not evaluate.
9. $y = x^2$, $x = y^2$ about the x -axis. Set up the integral, but do not evaluate.
10. $x = y^2 - 2y$, $x - 2y + 3 = 0$; about the x -axis. Set up the integral, but do not evaluate.
11. $y = \frac{1}{\sqrt{1+x^2}}$, $y = 1$, $x = 2$; about the y -axis. Set up the integral, but do not evaluate.
12. $y = \frac{1}{x^2 - 4x}$, $y = 0$, $x = 5$, $x = 10$; about the x -axis. Set up the integral, but do not evaluate.
13. $y = \sin x$, x -axis, $x = 0$, $x = \pi$; about the y -axis. Set up the integral, but do not evaluate.
14. $y = \begin{cases} \frac{3}{4} & 0 \leq x \leq 1 \\ \frac{3}{4}x^2 & 1 \leq x \leq 2 \end{cases}$ in the first quadrant about the x -axis.
15. $y = e^x$, $y = e$, y -axis; about the x -axis.
16. $y = e^x$, $y = e$, y -axis; about the y -axis.
17. $y = x^2$, $y = 1$, $x = 3$; about the x -axis.
18. $y = x^2$, $y = 1$, $x = 3$; about the x -axis.
19. $y = x^2 + 2$, $y = x + 8$; about the x -axis.
20. $y = 4x$, $y = 2x^2$; about the y -axis.
21. $y = x^3(2 - x)$, x -axis, first quadrant; about $x = 0$.
22. $f(x) = 5x$, $g(x) = x^2$, $x = 0$, $x = 3$; about the x -axis.
23. $f(x) = (x - 2)^2$ for $1 \leq x \leq 3$, x -axis; about the y -axis.
24. Set up, but do not evaluate, an integral for the volume of revolution obtained when the region bounded by $y = \arctan x$, x -axis, $x = 1$, is revolved about the y -axis.
25. Set up, but do not evaluate, an integral for the volume of revolution obtained when the region bounded by $y = \sqrt{x}$, $x = 1$, $x = 4$, $y = 0$ is revolved about the y -axis.
26. Set up, but do not evaluate, an integral for the volume of revolution obtained when the region bounded by $f(x) = x^2 + 1$, $g(x) = 2x + 4$, $x = 0$, in the first quadrant, is revolved about the y -axis.

Answers:

$$1. \int_0^2 \pi(4 - x^2)^2 dx = \frac{256\pi}{15}$$

$$2. \int_0^1 2\pi x(x - x^3) dx + \int_{-1}^0 2\pi(-x)(x^3 - x) dx = \frac{8\pi}{15}$$

$$3. \int_0^\pi \pi \sin^2 x dx$$

$$4. \int_0^1 \pi[(2x - x^2)^2 - (x^2)^2] dx$$

$$5. \int_0^2 \pi \left[(x - 2)^2 - \frac{1}{4}(x - 2)^4 \right] dx$$

$$6. \int_0^{\pi/4} \pi(\cos^2 x - \sin^2 x) dx + \int_{\pi/4}^{\pi/2} \pi(\sin^2 x - \cos^2 x) dx$$

$$7. \int_{-\pi/2}^0 \pi(-\sin y)^2 dy + \int_0^{\pi/2} \pi(\sin y)^2 dy$$

or $\int_{-1}^0 2\pi(-x)[\arcsin x - (-\frac{\pi}{2})] dx + \int_0^1 2\pi x[\frac{\pi}{2} - \arcsin x] dx$

$$8. \int_0^1 2\pi y(\sqrt{y} - y^2) dy$$

$$9. \int_0^1 \pi [(\sqrt{x})^2 - (x^2)^2] dx$$

$$10. \int_1^3 2\pi y [(2y - 3) - (y^2 - 2y)] dy$$

$$11. \int_0^2 2\pi x \left[1 - \frac{1}{\sqrt{1+x^2}} \right] dx$$

$$12. \int_5^{10} \pi \left(\frac{1}{x^2 - 4x} \right)^2 dx$$

$$13. \int_0^\pi 2\pi x \sin x dx$$

$$14. \int_0^1 \pi \left(\frac{3}{4} \right)^2 dx + \int_1^2 \pi \left(\frac{3x^2}{4} \right)^2 dx = \frac{81\pi}{20}$$

$$15. \int_0^1 \pi(e^2 - e^{2x}) dx = \frac{\pi}{2}(e^2 + 1)$$

$$16. \int_0^1 2\pi x(e - e^x) dx = \pi(e - 2)$$

$$17. \int_1^3 \pi [(x^2)^2 - 1^2] dx = \frac{232\pi}{5}$$

$$18. \int_1^9 2\pi y(3 - \sqrt{y}) dy = \frac{232\pi}{5}$$

$$19. \int_{-2}^3 \pi [(x + 8)^2 - (x^2 + 2)^2] dx = 250\pi$$

$$20. \int_0^2 2\pi x[4x - 2x^2] dx = \frac{16\pi}{3}$$

$$21. \int_0^2 2\pi x[x^3(2 - x)] dx = \frac{64\pi}{15}$$

$$22. \int_0^3 \pi [(5x)^2 - (x^2)^2] dx = \frac{882\pi}{5}$$

$$23. \int_1^3 2\pi x(x - 2)^2 dx = \frac{8}{3}\pi$$

$$24. \int_0^1 2\pi x \arctan x dx$$

$$25. \int_1^4 2\pi x\sqrt{x} dx$$

$$26. \int_0^3 2\pi x [(2x + 4) - (x^2 + 1)] dx$$