Să se calculeze derivatele următoarelor funcţii:

1) $f(x) = x^4 + 5x^3 - 8.$ 2) $f(x) = x^2 + \sqrt{x} - \sqrt[3]{x}.$

3) $f(x) = x\cos x.$ 4) $f(x) = \dfrac{x-1}{x^2+1}.$

5) $f(x) = \dfrac{\sin x}{2+\cos x}.$ 6) $f(x) = \ln \dfrac{x^2}{x+1}.$

7) $f(x) = \sqrt[3]{\dfrac{1-x^2}{1+x^2}}.$ 8) $f(x) = e^{x^2 \cos x}.$

R: Se obţine:

1) $f'(x) = 4x^3 + 15x^2.$ 2) $f'(x) = 2x + \dfrac{1}{2\sqrt{x}} - \dfrac{1}{3\left(\sqrt[3]{x}\right)^2}.$

3) $f'(x) = \cos x - x\sin x.$ 4) $f'(x) = -\dfrac{x^2-2x-1}{\left(x^2+1\right)^2}.$

5) $f'(x) = \dfrac{2\cos x + 1}{\left(2+\cos x\right)^2}.$ 6) $f'(x) = \dfrac{1}{x}\dfrac{x+2}{x+1}.$

7) $f'(x) = -\dfrac{4}{3}\dfrac{x}{\left(x^2+1\right)^2}\sqrt[3]{\left(\dfrac{x^2+1}{1-x^2}\right)^2}.$

8) $f'(x) = \left(2x\cos x - x^2\sin x\right)e^{x^2\cos x}.$

Să se calculeze derivatele următoarelor funcţii:

1) $f(x) = \ln\left(\sqrt{2\sin x + 1} + \sqrt{2\sin x - 1}\right).$

2) $f(x) = \dfrac{\sin x}{\cos^2 x} + \ln \dfrac{1 + \sin x}{\cos x}.$

3) $f(x) = \dfrac{x}{2}\sqrt{x^2 + k} + \dfrac{k}{2}\ln\left(x + \sqrt{x^2 + k}\right).$

4) $f(x) = 5\operatorname{sh}^3 \dfrac{x}{15} + 3\operatorname{sh}^5 \dfrac{x}{15}.$

5) $f(x) = e^x \operatorname{arctg} e^x - \ln\sqrt{1 + e^{2x}}.$

6) $f(x) = \dfrac{x^x}{e^x}(x\ln x - x - 1).$

7) $f(x) = \dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2}\arcsin\dfrac{x}{a}.$

8) $f(x) = \log_{e^2}\left(x^n + \sqrt{x^{2n} + 1}\right).$

R: Se obţine:

1) $f'(x) = \dfrac{\cos x}{\sqrt{(4\sin^2 x - 1)}}$. 2) $f'(x) = \dfrac{2}{\cos^3 x}$.

3) $f'(x) = \sqrt{x^2 + k}$. 4) $f'(x) = \operatorname{sh}^2 \dfrac{x}{15} \operatorname{ch}^3 \dfrac{x}{15}$.

5) $f'(x) = e^x \operatorname{arctg} e^x$. 6) $f'(x) = x^{x+1}e^{-x}(\ln x)(\ln x - 1)$.

7) $f'(x) = \sqrt{a^2 - x^2}$. 8) $f'(x) = \dfrac{nx^{n-1}}{2\sqrt{x^{2n} + 1}}$.

Să se calculeze derivatele următoarelor funcţii:

$$1)\ f(x)=\ln\frac{1+\sqrt{\sin x}}{1-\sqrt{\sin x}}+2\operatorname{arctg}\left(\sqrt{\sin x}\right).$$

$$2)\ f(x)=\frac{3}{4}\ln\frac{x^2+1}{x^2-1}+\frac{1}{4}\ln\frac{x-1}{x+1}+\frac{1}{2}\operatorname{arctg}x.$$

$$3)\ f(x)=\frac{1}{3}\ln(1+x)-\frac{1}{6}\ln\left(x^2-x+1\right)+\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}.$$

$$4)\ f(x)=3b^2\operatorname{arctg}\sqrt{\frac{x}{b-x}}-(3b+2x)\sqrt{bx-x^2}.$$

R: 1) $f'(x)=\dfrac{2}{\cos x\sqrt{\sin x}}$. 2) $f'(x)=\dfrac{x(x-3)}{x^4-1}$. 3) $f'(x)=\dfrac{1}{x^3+1}$.
4) $f'(x)=4x\sqrt{\dfrac{x}{b-x}}$.

Să se calculeze derivatele următoarelor funcţii:

1) $f(x) = -\dfrac{\arcsin x}{x} + \ln \dfrac{x}{1+\sqrt{1-x^2}}$.

2) $f(x) = \ln \sqrt{x^4+x^2+1} + \dfrac{2}{\sqrt{3}} \operatorname{arctg} \dfrac{2x^2+1}{\sqrt{3}}$.

3) $f(x) = \dfrac{x}{4\left(x^2+1\right)^2} + \dfrac{3x}{8\left(x^2+1\right)} + \dfrac{3}{8} \operatorname{arctg} x$.

4) $f(x) = \dfrac{5}{2}\sqrt{(2x^2+8x+1)} - \dfrac{13}{\sqrt{2}} \ln\left(\sqrt{2}\,(x+2) + \sqrt{(2x^2+8x+1)}\right)$.

R: Se obţine:

1) $f'(x) = \dfrac{\arcsin x}{x^2}$. 2) $f'(x) = \dfrac{2x^3+3x}{x^4+x^2+1}$.

3) $f'(x) = \dfrac{1}{\left(x^2+1\right)^3}$. 4) $f'(x) = \dfrac{5x-3}{\sqrt{2x^2+8x+1}}$.

Să se calculeze derivatele de ordinul doi ale următoarelor funcţii:

1) $f(x) = x^8 + 7x^6 - 5x + 4.$ 2) $f(x) = (\arcsin x)^2.$ 3) $f(x) = e^{x^2}.$
4) $f(x) = \ln\left(x + \sqrt{a^2 + x^2}\right).$ 5) $f(x) = \left(1 + x^2\right) \operatorname{arctg} x.$ 6) $f(x) = \sin^2 x.$

R: Se obţine:
1) $f''(x) = 56x^6 + 210x^4$. 2) $f''(x) = \dfrac{2}{1 - x^2} + \dfrac{2x}{\sqrt{(1 - x^2)^3}} \arcsin x.$
3) $f''(x) = 2e^{x^2} + 4x^2 e^{x^2}$. 4) $f''(x) = -\dfrac{x}{\sqrt{(a^2 + x^2)^3}}.$
5) $f''(x) = 2\operatorname{arctg} x + 2\dfrac{x}{x^2 + 1}$. 6) $f''(x) = 2\cos 2x.$

Să se calculeze derivatele parţiale ale următoarelor funcţii:

1) $f(x,y) = x^3 + y^3 - 3axy.$ 2) $f(x,y) = \dfrac{x-y}{x+y}.$

3) $f(x,y) = \sqrt{x^2 - y^2}.$ 4) $f(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}.$

5) $f(x,y) = \ln\left(x + \sqrt{x^2+y^2}\right).$ 6) $f(x,y) = \operatorname{arctg}\dfrac{y}{x}.$

7) $f(x,y) = e^{\sin\frac{y}{x}}.$ 8) $f(x,y) = \arcsin\sqrt{\dfrac{x^2-y^2}{x^2+y^2}}.$

R: Se obţine:

1) $f'_x(x,y) = 3x^2 - 3ay,\ f'_y(x,y) = 3y^2 - 3ax.$

2) $f'_x(x,y) = \dfrac{2y}{(x+y)^2},\ f'_y(x,y) = \dfrac{-2x}{(x+y)^2}.$

3) $f'_x(x,y) = \dfrac{x}{\sqrt{x^2-y^2}},\ f'_y(x,y) = \dfrac{-y}{\sqrt{x^2-y^2}}.$

4) $f'_x(x,y) = \dfrac{y^2}{\sqrt{(x^2+y^2)^3}},\ f'_y(x,y) = \dfrac{x^2}{\sqrt{(x^2+y^2)^3}}.$

5) $f'_x(x,y) = \dfrac{1}{\sqrt{x^2+y^2}},\ f'_y(x,y) = \dfrac{y}{\sqrt{x^2+y^2}\left(x+\sqrt{x^2+y^2}\right)}.$

6) $f'_x(x,y) = -\dfrac{y}{x^2+y^2},\ f'_y(x,y) = \dfrac{x}{x^2+y^2}.$

7) $f'_x(x,y) = -\dfrac{y}{x^2}e^{\sin\frac{y}{x}}\cos\dfrac{y}{x},\ f'_y(x,y) = \dfrac{1}{x}e^{\sin\frac{y}{x}}\cos\dfrac{y}{x}.$

8) $f'_x(x,y) = \dfrac{xy\sqrt{2}}{(x^2+y^2)\sqrt{x^2-y^2}},\ f'_y(x,y) = -\dfrac{x^2\sqrt{2}}{(x^2+y^2)\sqrt{x^2-y^2}}.$

Să e calculeze derivatele parţiale ale următoarelor funcţii:

1) $f(x,y,z) = x^3y^2z + 2x - 3y + z + 5.$ 2) $f(x,y,z) = (xy)^z.$
3) $f(x,y,z) = \sqrt{x^2+y^2+z^2}.$ 4) $f(x,y,z) = z^{xy}.$

R: Se obţine:
1) $f'_x(x,y,z) = 3x^2y^2z + 2,\ f'_y(x,y,z) = 2x^3yz - 3,\ f'_z(x,y,z) = x^3y^2 + 1.$
2) $f'_x(x,y,z) = \frac{z}{x}(xy)^z,\ f'_y(x,y,z) = \frac{z}{y}(xy)^z,\ f'_z(x,y,z) = (xy)^z\ln(xy).$
3) $f'_x(x,y,z) = \frac{x}{\sqrt{x^2+y^2+z^2}},\ f'_y(x,y,z) = \frac{y}{\sqrt{x^2+y^2+z^2}},$
$f'_z(x,y,z) = \frac{z}{\sqrt{x^2+y^2+z^2}}.$
4) $f'_x(x,y,z) = y\ z^{xy}\ln z,\ f'_y(x,y,z) = xz^{xy}\ln z,\ f'_z(x,y,z) = \frac{xy}{z}z^{xy}.$

Să se arate că funcţiile date mai jos satisfac egalităţile scrise în dreptul lor:

$$1)\ z=\ln\left(x^2+xy+y^2\right),\quad x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2.$$

$$2)\ z=xy+xe^{\frac{y}{x}},\quad x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=xy+z.$$

$$3)\ u=(x-y)(y-z)(z-x),\quad \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$$

$$4)\ u=x+\frac{x-y}{y-z},\quad \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=1.$$

$$5)\ u=\ln\left(x^3+y^3+z^3-3xyz\right),\quad \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{1}{x+y+z}.$$

Să se determine punctele de extrem ale funcţiilor:

1) $f(x,y) = x^3 + 3xy^2 - 15x - 12y$. 2) $f(x,y) = x^2 + xy + y^2 - 3x - 6y$.

3) $f(x,y) = \dfrac{1}{2}xy + (47 - x - y)\left(\dfrac{x}{3} + \dfrac{y}{4}\right)$. 4) $f(x,y) = x^3 + y^3 + 3xy$.

R: 1) Punctele staţionare sunt soluţiile sistemului:

$$\frac{\partial f}{\partial x} = 3(x^2 + y^2 - 5) = 0, \quad \frac{\partial f}{\partial y} = 6(xy - 2) = 0,$$

adică: $(2,1)$, $(-2,-1)$, $(1,2)$, $(-1,-2)$. Derivatele de ordinul doi sunt:

$$\frac{\partial^2 f}{\partial x^2} = 6x, \quad \frac{\partial^2 f}{\partial x \partial y} = 6y, \quad \frac{\partial^2 f}{\partial y^2} = 6x.$$

In punctul $(2,1)$, $\Delta_1 = 12 > 0$, $\Delta_2 = 108 > 0$, $(2,1)$ este un punct de minim, $f(2,1) = -28$. In punctul $(-2,-1)$, $\Delta_1 = -12 < 0$, $\Delta_2 = 108 > 0$, $(-2,-1)$ este un punct de maxim, $f(-2,-1) = 28$. In punctele $(1,2)$, $(-1,-2)$, $\Delta_2 = -108 < 0$. Nu sunt puncte de extrem.

2) Un punct staţionar: $(0,3)$. $\Delta_1 = 2 > 0$, $\Delta_2 = 3 > 0$. Punctul $(0,3)$ este un punct de minim şi $f_{\min} = f\,(0,3) = -9$.

3) Un punct staţionar: $(21,20)$. $\Delta_1 = -\dfrac{2}{3} < 0$, $\Delta_2 = \dfrac{47}{144} > 0$. Punctul $(21,20)$ este un punct de maxim şi $f_{\max} = f\,(21,20) = 282$.

4) Un punct staţionare: $(0,0)$, $(-1,-1)$. Punctul $(0,0)$ nu este punct de extrem. Punctul $(-1,-1)$ este un punct de maxim şi $f_{\max} = f\,(-1,-1) = 1$.

Să se integreze următoarele ecuaţii diferenţiale cu variabile separabile:

1) $x\,dt + t\,dx = 0$.
2) $tx' - x = x^3$.
3) $txx' = 1 - t^2$.
4) $\operatorname{tg} t \sin^2 x\,dt + \operatorname{ctg} x \cos^2 t\,dx = 0$.
5) $dx = \left(t^2+1\right)\left(x^2+1\right)dt$.
6) $\left(t^2-1\right)x' - tx = 0$.
7) $x' + \sin(t+x) = \sin(t-x)$.
8) $x' = \operatorname{sh}(t+x) + \operatorname{sh}(t-x)$.

R: 1) Ecuaţia se mai scrie, separând variabilele: $\dfrac{1}{t}\,dt + \dfrac{1}{x}\,dx = 0$. De unde $\ln|t| + \ln|x| = \ln|C|$, sau $tx = C$.

2) $t^2\left(1+x^2\right) = Cx^2$. 3) $x^2 - 2\ln t + t^2 = C$. 4) $\operatorname{ctg}^2 x = \operatorname{tg}^2 t + C$.

5) $x = \operatorname{tg}\left(\dfrac{1}{3}t^3 + t + C\right)$. 6) $x^2 = C\left(t^2-1\right)$. 7) $\ln\left|\operatorname{tg}\dfrac{x}{2}\right| + 2\sin t = C$.

8) $x = \ln[\operatorname{tg}(\operatorname{ch} t + C)]$.

Să se integreze următoarele ecuaţii diferenţiale cu variabile separabile, cu condiţiile iniţiale precizate:

1) $(1+e^t)\,xx' = e^t$, *cu* $x(0)=1$.
2) $\left(1+e^{2t}\right)x^2dx = e^t dt$, *cu* $x(0)=0$.
3) $x' + \cos(t+2x) = \cos(t-2x)$, *cu* $x(0)=\dfrac{\pi}{4}$.
4) $e^{1+t^2}\operatorname{th} x\,dt - \dfrac{1}{t-1}e^{2t}dx = 0$, *cu* $x(1)=\dfrac{\pi}{2}$.
5) $x' = e^{t+x}+e^{t-x}$, *cu* $x(0)=0$.
6) $x(t+2)\,dt + t(x-1)\,dx = 0$, *cu* $x(1)=1$.
7) $t\left(x^6+1\right)dt + x^2\left(t^4+1\right)dx = 0$, *cu* $x(0)=1$.

R: 1) $x^2 = 1+2\ln\dfrac{1+e^t}{2}$. 2) $\dfrac{1}{3}x^3+\dfrac{\pi}{4} = \operatorname{arctg} e^t$. 3) $\ln|\operatorname{tg} x| = 4(1-\cos t)$.
4) $\ln\sin^2 x = e^{(x-1)^2}-1$. 5) $x = \ln\left(e^t+\dfrac{\pi}{4}-1\right)$. 6) $t+x+2\ln t-\ln x = 2$.
7) $3\operatorname{arctg} t^2 + 2\operatorname{arctg} x^3 = \dfrac{\pi}{2}$.

Să se integreze următoarele ecuaţii diferenţiale cu variabile separabile:

1) $(\cos t - \sin t + 1)\, x' = \cos x - \sin x - 1.$ 2) $2t\sqrt{1-x^2} = x'\left(1+t^2\right).$
3) $e^t \sin^3 x + \left(1+e^{2t}\right)(\cos x)\, x' = 0.$ 4) $x^2 (\sin t) + \left(\cos^2 t\right)(\ln x)\, x' = 0.$
5) $x' = \sin(t-x).$ 6) $x + tx' = a(1+tx).$
7) $(tx-1)^2\, tx' + \left(t^2x^2+1\right)x = 0.$ 8) $t\sqrt{1+x^2} + x\sqrt{1+t^2}\, x' = 0.$

R: 1) $\operatorname{tg}\frac{x}{2} = C\left(1+\operatorname{tg}\frac{x}{2}\right)\left(1-\operatorname{tg}\frac{t}{2}\right)$. 2) $x = \sin\left[C\ln\left(1+t^2\right)\right]$.
3) $\operatorname{arctg} e^t = \frac{1}{2\sin^2 x} + C$. 4) $x = (1+\ln x + Cx)\cos t$. 5) $t + C = \operatorname{ctg}\left(\frac{x-t}{2}+\frac{\pi}{4}\right)$.
6) $1 + tx = Ce^{at}$. 7) Cu schimbarea de funcţie $u = tx$, obţinem $x^2 = Ce^{tx - \frac{1}{tx}}$.
8) $\sqrt{1+t^2} + \sqrt{1+x^2} = C$.

Să se integreze următoarele ecuaţii liniare de ordinul întâi:

1) $x' - x\operatorname{ctg} t + 2t\sin t = 0.$

2) $\left(1+t^2\right)x' + x - \operatorname{arctg} t = 0.$

3) $x' + ax - be^{pt} = 0,\ a, b, p \in \mathbf{R}.$

4) $tx' - \dfrac{1}{t+1}x - t + 1 = 0.$

5) $\left(t^2-1\right)^{\frac{3}{2}} x' t^3 + 3tx\sqrt{t^2-1} = 0.$

6) $\sqrt{1+t^2}\,x' + x + t - \sqrt{1+t^2} = 0.$

7) $t\left(t^3+1\right)x' + \left(2t^3-1\right)x = \dfrac{t^3-2}{t}.$

8) $x' - \dfrac{n}{t+1}x = e^t\left(t+1\right)^n,\ n \in \mathbf{N}.$

R: 1) $x(t) = t^2\sin t + C\sin t.$ 2) $x(t) = \operatorname{arctg} t - 1 + Ce^{-\operatorname{arctg} t}.$

3) $x(t) = \dfrac{b}{a+p}e^{pt} + Ce^{-at}$, pentru $p \neq -a$ şi $x(t) = (bt + C)e^{-at}$, pentru $p = -a$.

4) $x(t) = t + 1 + \dfrac{C}{t+1}e^t.$ 5) $x(t) = \dfrac{1}{4}\left(C - t^4\right)\left(t^2-1\right)^{-\frac{3}{2}}.$

6) $x(t) = \dfrac{1}{t+\sqrt{1+t^2}}\left(\ln\left(t+\sqrt{1+t^2}\right) + C\right).$

7) $x(t) = \dfrac{1}{t} + \dfrac{Ct}{t^3+1}.$ 8) $x(t) = \left(e^t + C\right)(t+1)^n.$

Să se integreze următoarele ecuaţii liniare de ordinul întâi, cu condiţiile iniţiale precizate:

1) $x' = \dfrac{x}{1-t^2} - t - 1,\ cu\ x(0) = 0.$ 2) $tx' + x = e^t,\ cu\ x(a) = b\ (a \neq 0).$

3) $tx' - nx = t^{n+1}\ln t,\ cu\ x(1) = 0.$ 4) $x'\cos^2 t + x - \operatorname{tg} t = 0\ cu\ x(0) = 0.$

5) $tx' - nx = t^{n+1}e^t,\ cu\ x(1) = 1.$ 6) $tx' + \left(2t^2 - 1\right)x = 2t^2 - 1,\ cu\ x(1) = 1 - \dfrac{1}{e}.$

R: 1) $x(t) = \left(\dfrac{1}{2}t\sqrt{1-t^2} + \dfrac{1}{2}\arcsin t\right)\sqrt{\dfrac{1+t}{1-t}}$. 2) $x(t) = \dfrac{1}{t}\left(e^t - e^a + ab\right)$.
3) $x(t) = \dfrac{1}{4}\left(t^n - t^{n+2}\right) + \dfrac{1}{2}t^{n+2}\ln|t|$. 4) $x(t) = -1 + \operatorname{tg} t + e^{-\operatorname{tg} t}$.
5) $x(t) = t^n\left(e^t - e + 1\right)$. 6) $x(t) = 1 - te^{-t^2}$.

Să se găsească soluţiile generale ale ecuaţiilor diferenţiale liniare de ordinul al doilea cu coeficienţi constanţi:

1) $x'' - 5x' + 6x = 0.$ 2) $x'' - 9x = 0.$ 3) $x'' - x' = 0.$
4) $x'' + x = 0.$ 5) $x'' - 2x' + 2x = 0.$ 6) $x'' + 4x' + 13x = 0.$

R: 1) Ecuaţia caracteristică $r^2 - 5r + 6 = 0$, are rădăcinile $r_1 = 2$, $r_2 = 3$. Soluţia generală este $x(t) = C_1e^{2t} + C_2e^{3t}$. 2) $x(t) = C_1e^{-3t} + C_2e^{2t}$. 3) $x(t) = C_1 + C_2e^t$.
4) $x(t) = C_1 \cos t + C_2 \sin t$. 5) $x(t) = e^t (C_1 \cos t + C_2 \sin t)$. 6) $x(t) = e^{-2t}(C_1 \cos 3t + C_2 \sin 3t)$.

Să se găsească soluţiile generale ale ecuaţiilor diferenţiale liniare cu coeficienţi constanţi de ordinul al doilea, neomogene:

1) $2x'' - x' - x = 4te^{2t}$. 2) $x'' - 2x' + x = te^t$. 3) $x'' + x = t\sin t$.
4) $x'' + x = t^2 + t$. 5) $x'' + x' = t - 2$. 6) $x'' - x = te^{2t}$.
7) $x'' - 7x' + 6x = \sin t$. 8) $x'' + 4x = t\sin 2t$. 9) $x'' + 3x' + 2x = t\sin t$.

R: 1) Se caută o soluţie particulară de forma: $x^*(t) = e^{2t}(At + B)$. Se obţine

$$x(t) = C_1e^t + C_2e^{-\frac{t}{2}} + e^{2t}\left(\frac{4}{5}t - \frac{28}{25}\right).$$

2) Se caută o soluţie particulară de forma: $x^*(t) = t^2e^t(At + B)$. Se obţine

$$x(t) = (C_1 + C_2t)\,e^t + \frac{1}{6}t^3e^t.$$

3) Se caută o soluţie particulară de forma: $x^*(t) = t[(At + B)\cos t + (Ct + D)\sin t]$. Se obţine

$$x(t) = C_1\cos t + C_2\sin t - \frac{t^2}{4}\cos t + \frac{t}{4}\sin t.$$

4) $x\,(t) = -2 + t + t^2 + C_1\cos t + C_2\sin t$.

5) $x\,(t) = C_1 + C_2e^{-t} - 3t + \frac{1}{2}t^2$.

6) $x\,(t) = C_1e^t + C_2e^{-t} + \frac{1}{3}\left(t - \frac{4}{3}\right)e^{2t}$.

7) $x\,(t) = C_1e^t + C_2e^{6t} + \frac{7}{74}\cos t + \frac{5}{74}\sin t$.

8) $x\,(t) = C_1\cos 2t + C_2\sin 2t - \frac{1}{8}t^2\cos 2t + \frac{1}{16}t\sin 2t + \frac{1}{64}\cos 2t$.

9) $x\,(t) = C_1e^{-2t} + C_2e^{-t} + \left(-\frac{3}{10}t + \frac{17}{50}\right)\cos t + \left(\frac{1}{10}t + \frac{3}{25}\right)\sin t$.

Să se găsească soluţiile generale ale ecuaţiilor diferenţiale liniare cu coeficienţi constanţi de ordinul al doilea, neomogene:

$$1)\ x'' + x' = 4t^2e^t. \qquad 2)\ x'' + 10x' + 25x = 4e^{-5t}.$$
$$3)\ x'' - 6x' + 9x = 25e^t \sin t. \qquad 4)\ x'' + 2x' + 5x = e^{-t}\cos 2t.$$

R: Avem:

1) $x(t) = C_1 + C_2e^{-t} + \left(2t^2 - 6t + 7\right)e^t$.

2) $x(t) = (C_1 + C_2t)\,e^{-5t} + 2t^2e^{-5t}$.

3) $x(t) = C_1e^{3t} + C_2e^{3t}t + (4\cos t + 3\sin t)\,e^t$.

4) $x(t) = C_1e^{-t}\cos 2t + C_2e^{-t}\sin 2t + \dfrac{1}{4}te^{-t}\sin 2t$.