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On Lie solvable group algebras

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The group algebra FG of a group G over a field F may be considered as a Lie algebra with the Lie product $[x, y] = xy - yx$, where $x, y \in FG$. Denote by $[X, Y]$ the additive subgroup generated by all Lie products $[x, y]$ with $x \in X$ and $y \in Y$. We say that FG is *strongly Lie solvable* if the *strong Lie derived series* $\delta^{(n)}(FG) = [\delta^{(n-1)}(FG), \delta^{(n-1)}(FG)]FG$ with $\delta^{(0)}(FG) = FG$ vanishes for some integer n; the minimal such integer is called the *strong Lie derived length* of FG and is denoted by $\mathrm{dl}^L(FG)$. Furthermore, FG is called *Lie solvable*, if some of the terms of the *Lie derived series* $\delta^{[n]}(FG) = [\delta^{[n-1]}(FG), \delta^{[n-1]}(FG)]$ with $\delta^{[0]}(FG) = FG$ contain only the zero element. Denote by $\mathrm{dl}_L(FG)$ the minimal element of the set $\{m \in \mathbb{N} \mid \delta^{[m]}(FG) = 0\}$, which is said to be the *Lie derived length* of FG.

Although there exist criteria for both Lie and strongly Lie solvability of group algebras, the exact values of $\mathrm{dl}_L(FG)$ and $\mathrm{dl}^L(FG)$ are known only in a few cases. The characterization of groups satisfying $\mathrm{dl}_L(FG) = \mathrm{dl}^L(FG)$ is also an open problem. This talk will be devoted to summarize our related results on this topic. Furthermore, we will have a try to extend the concept of Lie solvability for subsets of group algebras, and determine the Lie derived length of some well-known subsets.