Scientific report

on the project

Transport Phenomena in Nanofluids and Nanofluids Saturated Porous Media

(code PN-II-RU-TE-2011-3-0013)

***period** October 2011 – October 2014*

Objectives:

O1. Study of convective heat transfer in nanofluids using boundary layer approximation
O2. Study of convective heat transfer in nanofluids saturated porous media using boundary layer approximation
O3. Study of convective heat transfer in channels and enclosures filed with nanofluids
O4. Study of convective heat transfer in channels and enclosures filed with nanofluid saturated porous media
O5. Study the effects of variable physical properties and heat generation on convective heat transfer in nanofluids.

In the period 5.10.2011-4.10.2014 were published in ISI Journals the following papers:

1. T. Grosan, Thermal dispersion effect on fully developed free convection of nanofluids in a vertical channel, Sains Malaysiana, Vol. 40(12), pp. 1429–1435, 2011. Factor de impact: 0.408, (partially supported from the grant –PN-II-RU-TE-2011-3-0013)
2. T. Grosan, I. Pop, Fully Developed Mixed Convection in a Vertical Channel Filled by a Nanofluid, Journal of Heat Transfer (ASME), Volume 134, Issue 8, 082501 (5 pages), 2012. Factor de impact: 1.83, (partially supported from the grant –PN-II-RU-TE-2011-3-0013)
3. A. V. Rosca, N. C. Rosca, T. Grosan, I. Pop, Non-Darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media, International Communications in Heat and Mass Transfer, Vol. 39 pp. 1080-1085, 2012. Factor de impact: 2.208
4. N. C. Rosca, T. Grosan, I. Pop, Stagnation-point Flow and Mass Transfer with Chemical Reaction Past a Permeable Stretching/shrinking Sheet in a Nanofluid, Sains Malayesiana, Vol. 41, pp. 1271–1279, 2012. Factor de impact: 0.408
5. A.V. Rosca, I. Pop, Flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip, Intrnational Journal of Heat and Mass Transfer, Vol. 60, pp. 355-364, 2013. Factor de impact: 2.315
6. A V. Rosca, I. Pop, Mixed Convection Stagnation-Point Flow Past a Vertical Flat Plate With a Second Order Slip, Journal of Heat Transfer (ASME), Vol. 136, Issue 1, 012501 (8 pages), 2014. Factor de impact: 2.055
7. T. Grosan, J.H. Merkin, I. Pop, Mixed convection boundary-layer flow on a horizontal flat surface with a convective boundary condition, Meccanica, Vol. 48, pp. 2149-2158, 2013. Factor de impact: 1.747
8. R. Trîmbiţaş, T. Grosan, I. Pop, Mixed convection boundary layer flow along vertical thin needles in nanofluids, International Journal of Numerical Methods for Heat and Fluid Flow, Vol. 24, pp. 579 – 594, 2014. Factor de impact: 0.919
9. F.O. Pătrulescu, T. Groşan, I. Pop, Mixed convection boundary layer flow from a vertical truncated cone in a nanofluid Int. Journal of Numerical Methods for Heat and Fluid Flow, Vol. 24, pp. 1175-1190, 2014. Factor de impact: 0.919
10. N.C. Rosca, A.V. Rosca, T. Grosan, I. Pop, Mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium saturated by a nanofluid: Darcy-Ergun model, International Journal of Numerical Methods for Heat and Fluid Flow, Vol. 24, pp. 970-987, 2014. Factor de impact: 0.919
11. Rosca, Natalia C.; Rosca, Alin V.; Pop, Ioan, Stagnation point flow and heat transfer over a non-linearly moving flat plate in a parallel free stream with slip, COMMUNICATIONS IN NONLINEAR SCIENCE AND NUMERICAL SIMULATION Volume: 19 Issue: 6 Pages: 1822-1835, 2014. Factor de impact: 2.569
12. M.A. Sheremet, T. Grosan, I. Pop, Free Convection in Shallow and Slender Porous Cavities Filled by a Nanofluid Using Buongiorno's Model, JOURNAL OF HEAT TRANSFER-TRANSACTIONS OF THE ASME, Vol. 136, Article Number: 082501, DOI: 10.1115/1.4027355, 2014. Factor de impact: 2.055

In the period 5.10.2011-15.12.2013 were accepted or sent for publication in ISI Journals the following papers:

13. Natalia C. Rosca, Alin V. Rosca, John H. Merkin and Ioan Pop, Mixed convection boundary-layer flow near the lower stagnation point of a horizontal circular cylinder with a second-order wall velocity condition and a constant surface heat flux, IMA Journal of Applied Mathematics (2013) Page 1 of 21, doi:10.1093/imamat/hxt045, accepted, in press
14. ALIN V. ROSCA, MD. J. UDDIN, IOAN POP, BOUNDARY LAYER FLOW OVER A MOVING VERTICAL FLAT PLATE WITH CONVECTIVE THERMAL BOUNDARY CONDITION, Bulletin of the Malaysian Mathematical Sciences Society, accepted, February, 2014.
15. R. Trîmbiţaş, T. Grosan, I. Pop, Mixed convection boundary layer flow past a vertical flat plate in a nanofluid: case of prescribed wall heat flux, Applied Mathematics and Mechanics, accepted.
16. M.A. Sheremet, T. Groşan, I. Pop, Steady-state free convection in right-angle porous trapezoidal cavity filled by a nanofluid: Buongiorno's mathematical model, European Journal of Mechanics - B/Fluids, sent for publication.

In the period 5.10.2011-4.10.2014 were published the following books:

17. T. Grosan, MODELAREA MATEMATICĂ A FENOMENELOR CONVECTIVE ÎN MEDII POROASE, (ISBN 978-606-17-0263-3), Casa Cărţii de Ştiinţă Cluj-Napoca, 2012.
18. A. V. Roșca, METODE DE SIMULARE MONTE CARLO CU APLICAȚII ÎN ECONOMIE, (ISBN 978-973-595-548-9), Casa Cărţii de Ştiinţă Cluj-Napoca, 2013.

In the period 5.10.2011-4.10.2014 were published in conferences volumes the following papers:

19. T.Grosan, C. Revnic, I. Pop, Free convection in a porous cavity filled with nanofluids, Latest Trends in Environmental and Manufacturing Engineering (Proceedings of the 5th WSEAS International Conference on Environmental and Geological Science and Engineering, Vienna, November 10-12, 2012), pp. 187-192.
20. C. Revnic, T. Grosan, I. Pop, D.B. Ingham, Free convection heat transfer in a square cavity filled with a porous medium saturated by a water-based nanofluid, Proceedings of 5th International Conference on Applications of Porous Media 2013, August 25-28, Cluj-Napoca, Romania, ISSN 978-973-595-546-5, pp.349-357.
21. N.C. Rosca, A.V. Rosca, I. Pop, Mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with water at 4?C with a convective boundary condition: opposing flow case, Proceedings of 5th International Conference on Applications of Porous Media 2013, August 25-28, Cluj-Napoca, Romania, ISSN 978-973-595-546-5, pp.359-369
22. D.A. Filip, R. Trimbitas, I. Pop, Fully developed assisting mixed convection flow through a vertical porous channel with an anisotropic permeability: Case of heat flux, Proceedings of 5th International Conference on Applications of Porous Media 2013, August 25-28, Cluj-Napoca, Romania, ISSN 978-973-595-546-5, pp.145-156.

In period 5.10.2011-4.10.2014 the research team members have participated to the following conferences:

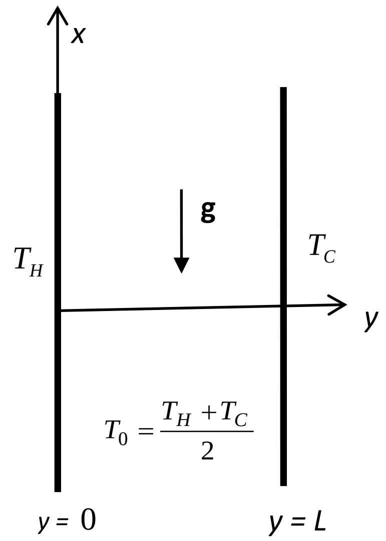
23. F. Patrulescu, M. Barboteu, A. Ramadan, On the behavior of a the solution to a contact problem with memory term, International Conference on Fixed Point Theory and its Applications, 9-12 June 2012, Cluj-Napoca, Romania
24. F. Patrulescu, A. Farcas, Analysis of a viscoplastic frictionless contact problem, XI-eme Colloque Franco-Roumain de Mathemtiques Appliques, 23 – 30 August 2012, Bucuresti, Romania
25. RADU TRÎMBIȚAȘ and TEODOR GROŞAN, Free convection heat transfer in a square cavity filled with a porous medium saturated by a water-based nanofluid in the presence of the internal heat generation, 3rd International Eurasian Conference on Mathematical Sciences & Applications, 25-28 August 2014, Vienna, Austria
26. R. Trîmbițaș and T. Groșan, Fully developed fluid flow and heat transfer in a nanofluid saturated porous medium with internal heat generation, 3rd International Eurasian Conference on Mathematical Sciences & Applications, 25-28 August 2014, Vienna, Austria

In the period 5.10.2011-4.10.2014 the following research visits were done:

27. Patrulescu Flavius, 23/07/2012 - 05/08/2012, Université de Perpignan, Perpignan, France
28. Pop Serban, 11/02/2013 - 15/02/2013, Universitatea Babes-Bolyai, Cluj-Napoca, Romania
29. Grosan Teodor,03/11/2013 - 08/11/2013, University of Leeds, Leeds, United Kingdom

Paper 1. Thermal dispersion effect on fully developed free convection of nanofluids in a vertical channel

The effect of thermal dispersion on the steady free convection flow of a nanofluid in a vertical channel is investigated numerically using a single phase model. Considering the laminar and fully developed flow regime a simplified mathematical model is obtained:



Geometry of the problem and co-ordinate system

$$\frac{d^2U}{dY^2} + \lambda_\phi \theta = 0$$

$$\frac{d}{dY}\left[\left(k_\phi + C\phi \Pr Gr \sqrt{U^2}\right)\frac{d\theta}{dY}\right] = 0$$

$$U(0) = 0, \quad U(1) = 0, \quad \theta(0) = 1, \quad \theta(1) = -1$$

where

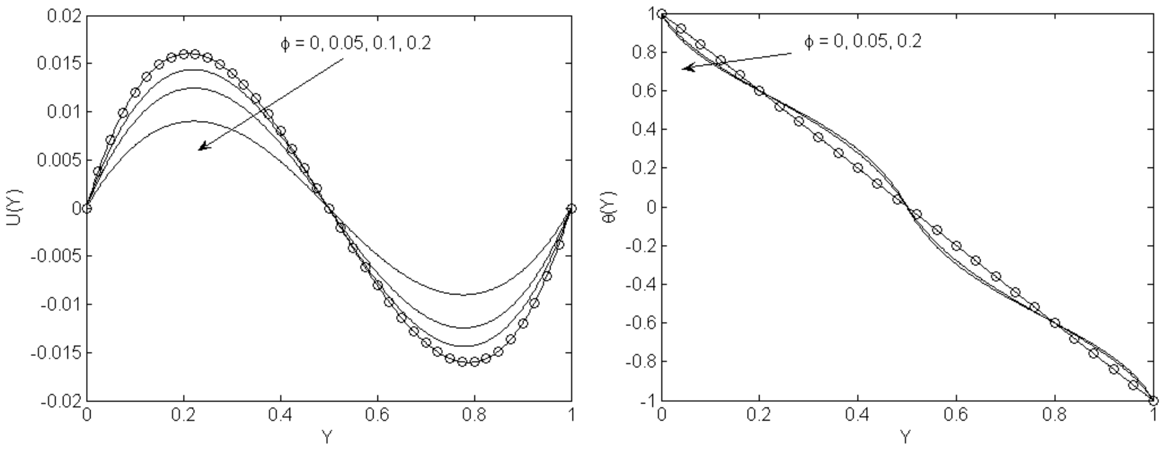
$$\lambda_\phi = (1-\phi)^{2.5}\left[\phi \frac{\rho_s}{\rho_f}\frac{\beta_s}{\beta_f} + (1-\phi)\right], k_\phi = \frac{k_{nf}/k_f}{(1-\phi) + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f}}$$

$$\Pr = \nu_f/\alpha_f \text{ și } Gr = g\beta_f (T_H - T_0)L^3/\nu_f{}^2$$

In the particular cases when solid phase and thermal dispersion effects are neglected the problem was solved analytically. The numerical solution is shown to be in excellent agreement with the close form analytical solution.

Nanoparticles	C	ϕ 0.05	ϕ 0.1	ϕ 0.2
Cu	0	2.314266	2.663273	3.491415
	0.1	2.320430	2.673790	3.506350
	0.2	2.326589	2.684295	3.521265
	0.3	2.332744	2.694787	3.536160
	0.4	2.338893	2.705266	3.551037
Al_2O_3	0	2.300996	2.633786	3.417282
	0.1	2.306959	2.643610	3.430207
	0.2	2.312917	2.653422	3.443117
	0.3	2.318870	2.663222	3.456011
	0.4	2.324819	2.673012	3.468891
TiO_2	0	2.256304	2.535483	3.175914
	0.1	2.262265	2.545300	3.188825
	0.2	2.268221	2.555105	3.201721
	0.3	2.274172	2.564899	3.214600
	0.4	2.280118	2.574681	3.227463

Values of Nusselt number for Gr = 10, different volume fractions ϕ and different values of constant C



Dimensionless velocity and temperature profiles for Cu nanoparticles, *C* = 0.4 and different values of ϕ

Nusselt number enhancement with the Grashof number, volume fraction and thermal diffusivity constant increasing has been found.

Paper 2. Fully Developed Mixed Convection in a Vertical Channel Filled by a Nanofluid

The steady fully developed mixed convection flow between two vertical parallel plates with asymmetrical thermal and nanoparticle concentration conditions at the walls filled by a nanofluid is studied. The nanofluid model used in this paper takes into account the Brownian diffusion and the thermophoresis effects, and the analysis is based on analytical solutions.

Assuming the Oberbeck–Boussinesq approximation, the following three equations embody the momentum, thermal energy, and nanoparticle concentration:

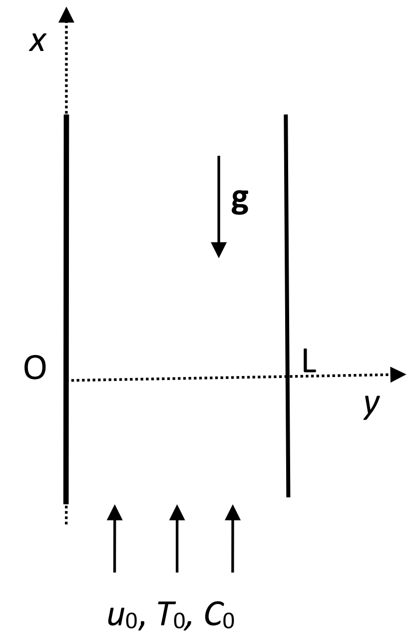
$$-\frac{d\,p}{d\,x}+\mu\frac{d^2u}{d\,y^2}+\left[(1-C_0)\rho_{f0}\,\beta(T-T_0)-(\rho_f-\rho_{f0})(C-C_0)\right]g=0$$

$$k\,\frac{d^2T}{d\,y^2}+(\rho c)_p\left[D_B\frac{d\,C}{d\,y}\frac{d\,T}{d\,y}+\left(\frac{D_T}{T_0}\right)\left(\frac{d\,T}{d\,y}\right)^2\right]=0$$

$$D_B\frac{d^2C}{d\,y^2}+\left(\frac{D_T}{T_0}\right)\frac{d^2T}{d\,y^2}=0$$

along with the boundary conditions:

$$\begin{array}{l} u=0,\quad T=T_1,\quad C=C_1\quad \text{at}\quad y=0 \\ u=0,\quad T=T_2,\quad C=C_2\quad \text{at}\quad y=L \end{array},\qquad \int_0^L u\,dy=Q$$



Physical model and coordinate system

Using the following transformations:

$$Y = y/L, \quad U(Y) = u(y)/u_0, \quad P(Y) = p(y)/(\rho u_0^2)$$
$$\theta(Y) = (T - T_0)/(T_2 - T_0), \quad \phi(Y) = (C - C_0)/(C_2 - C_0)$$
$$T_0 = (T_1 + T_2)/2, \; C_0 = (C_1 + C_2)/2.$$

the dimensionless modeling equations are given by:

$$\frac{d^2U}{dY^2} + \frac{Gr}{\mathrm{Re}}\theta - Nr\,\phi + \alpha = 0$$

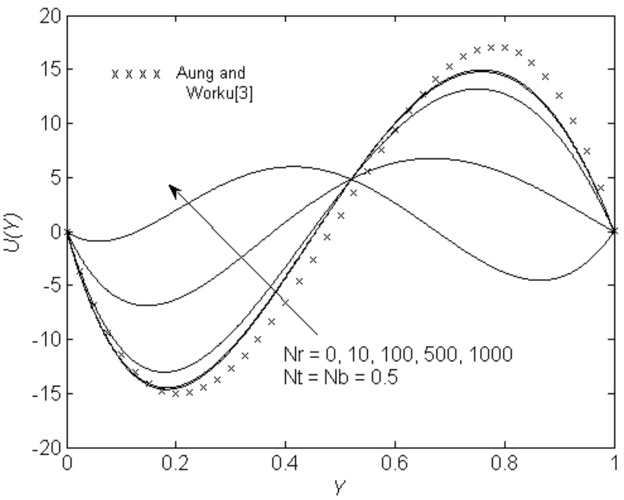
$$\frac{d^2\theta}{dY^2} + Nb\frac{d\theta}{dY}\frac{d\phi}{dY} + Nt\left(\frac{d\theta}{dY}\right)^2 = 0$$

$$\frac{d^2\phi}{dY^2} + \frac{Nt}{Nb}\frac{d^2\theta}{dY^2} = 0$$

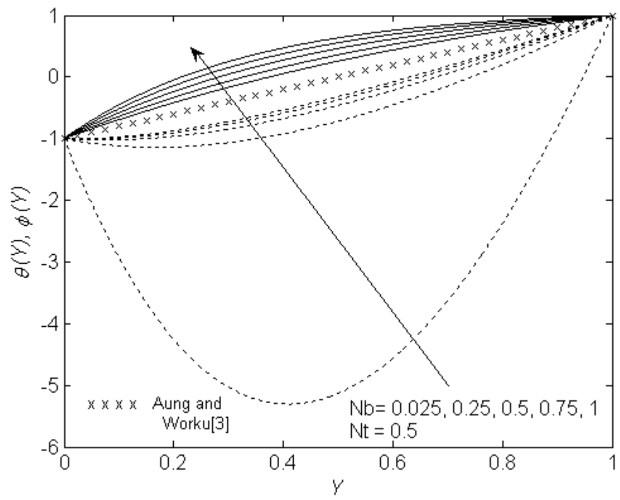
$$U(0) = 0, \quad \theta(0) = -1, \quad \phi(0) = -1$$
$$U(1) = 0, \quad \theta(1) = 1, \quad \phi(1) = 1 \quad , \quad \int_0^1 U\,dY = 1$$

Comparison between analytical and numerical results numeric

$\frac{Gr}{Re}$	Nr	Nt	Nb	α		$\theta'(0)$		$\phi'(0)$	
				analytic	numeric	analytic	Numeric	analytic	numeric
0	0	0	0.2	12	12.00000	2.42659	2.42647	2	1.99999
	0	0.2	0.2	12	12.00000	2.90554	2.90525	1.09445	1.09474
	5	0.2	0.2	11.209	11.20900	2.90554	2.90525	1.09445	1.09474
1000	0	0	0.2	-67.7723	-67.77230	2.42659	2.42647	2	1.99999
	0	0.2	0.2	-146.1991	-146.19916	2.90554	2.90525	1.09445	1.09474
	5	0.2	0.2	-146.9901	-146.99016	2.90554	2.90525	1.09445	1.09474



Velocity variation for $Nr = 0, 10, 100, 500, 1000$ and $Nt = Nb = 0.5$.



Temperature and concentration variation for $Nb = 0.025, 0.25, 0.5, 0.75, 1$ şi $Nt = 0.5$.

Thus, analytical expressions (using MATHEMATICA) for the fully developed velocity, temperature and nanoparticle concentration profiles as well as for the Nusselt and Sherwood numbers at the left wall of the channel are given. A numerical solution has been also obtained and compared with the analytical solution, the agreement being very good.

Paper 3. Non-Darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media

The problem of steady mixed convection boundary-layer flow over an impermeable horizontal flat plate embedded in a non-Darcy porous medium saturated by a nanofluid is numerically studied. The model used for the nanofluid incorporates only the effect of the volume fraction parameter. The surface of the plate is maintained at a constant temperature and a constant nano-particle volume fraction. The resulting governing partial differential equations are transformed into a set of two ordinary (similar) equations, which are solved using thebvp4c function from Matlab.

$$f'' + (1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)\,G\left[(f')^2\right]' = \frac{1}{2}(1-\phi)^{2.5}\left[1-\phi+\phi(\rho_s\beta_s)/(\rho_f\beta_f)\right]\lambda(\eta\theta'-\theta)$$

$$\frac{k_{nf}/k_f}{1-\phi+\phi(\rho C_p)_s/(\rho C_p)_f}\theta'' = \frac{1}{2}(\theta f' - f\theta')$$

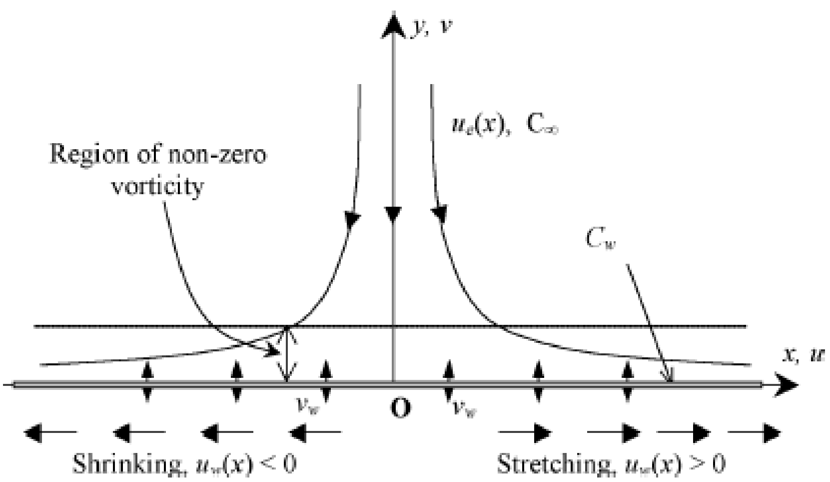
$$f(0)=0,\quad \theta(0)=1 \qquad \text{at} \quad \eta=0$$
$$f'(\eta)\to 1,\quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty.$$

We performed a multiple linear regression in order to obtain a correlation between Nur and φ, and G and λ:

$$Nur_{est} = 0.9337 + 1.2163\ \phi - 0.0827\ G + 0.1276\ \lambda$$

The coefficient of multiple determination is R2=0.9603 and the maximum relative error defined by $\varepsilon = \left|(Nur_{est} - Nur)/Nur\right|$ is $\varepsilon = 0.0424$.

Paper 4. Stagnation-point Flow and Mass Transfer with Chemical Reaction Past a Permeable Stretching/shrinking Sheet in a Nanofluid



A numerical study has been conducted to investigate the steady forced convection stagnation point-flow and mass transfer past a permeable stretching/shrinking sheet placed in a copper (Cu)- water based nanofluid. The external velocity of the fluid is $u_e(x)$, while the horizontal velocity of the plate $u_e(x) > 0$ (stretching) or $u_e(x) < 0$ (shrinking). The vertical velocity on the plate is constant, v_w. The system of partial differential equations is transformed, using appropriate transformations, into two ordinary differential equations:

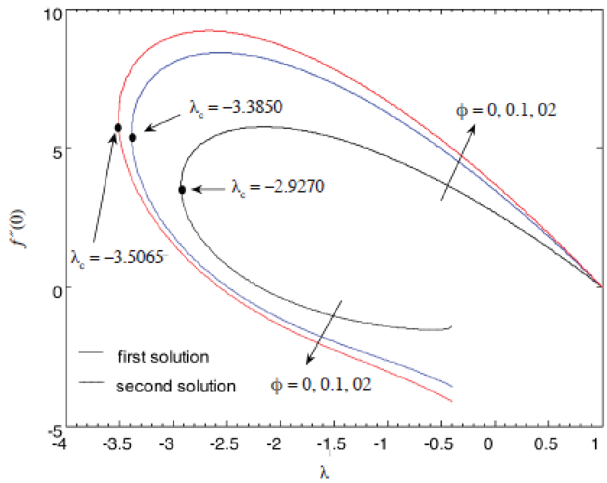
$$\frac{1}{(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)} f''' + ff'' - f'^2 + 1 = 0$$

$$\theta'' + Sc\, f\theta' - Sc\,\beta\theta = 0,$$

$$f(0) = s,\ f'(0) = \lambda,\ \theta(0) = 1 \ \text{at}\ \eta = 0$$
$$f'(\eta) \to 1,\ \theta(\eta) \to 0 \ \text{as}\ \eta \to \infty,$$

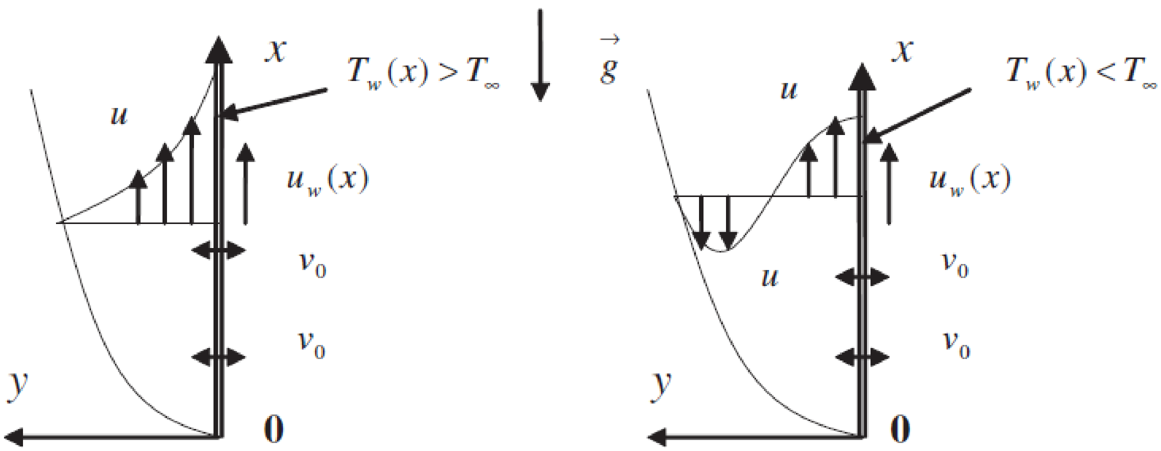
which are solved numerically using bvp4c function from Matlab. The results are obtained for the reduced skin-friction and reduced Sherwood number as well as for the velocity and concentration profiles for some values of the governing parameters.

These results indicate that dual solutions exist for the shrinking sheet case ($\lambda < 0$). In the next figure one can see the dual solutions for $\lambda > \lambda_{critic}$.



Paper 5. Flow and heat transfer over a permeable stretching/shrinking sheet with a second order slip

Steady flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip is investigated using a second order slip flow model. The governing equations are:



(a) Assisting flow (b) Opposing flow

$$
\begin{aligned}
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\\
&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+g\beta_T(T-T_\infty)\\
&\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\\
&\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)
\end{aligned}
$$

along with the boundary conditions

$$
\begin{aligned}
&t<0: \quad v=v_0, \quad u=0, \quad T=T_\infty \quad \text{for any } x,y\\
&t\geqslant 0: \quad v=v_0, \quad u=u_w(x)=cx+u_{\text{slip}}(x),\\
&\qquad T=T_w(x)=T_\infty+T_0x \quad \text{at } y=0\\
&\qquad u=0, \quad T=T_\infty \quad \text{as } y\to\infty
\end{aligned}
$$

where

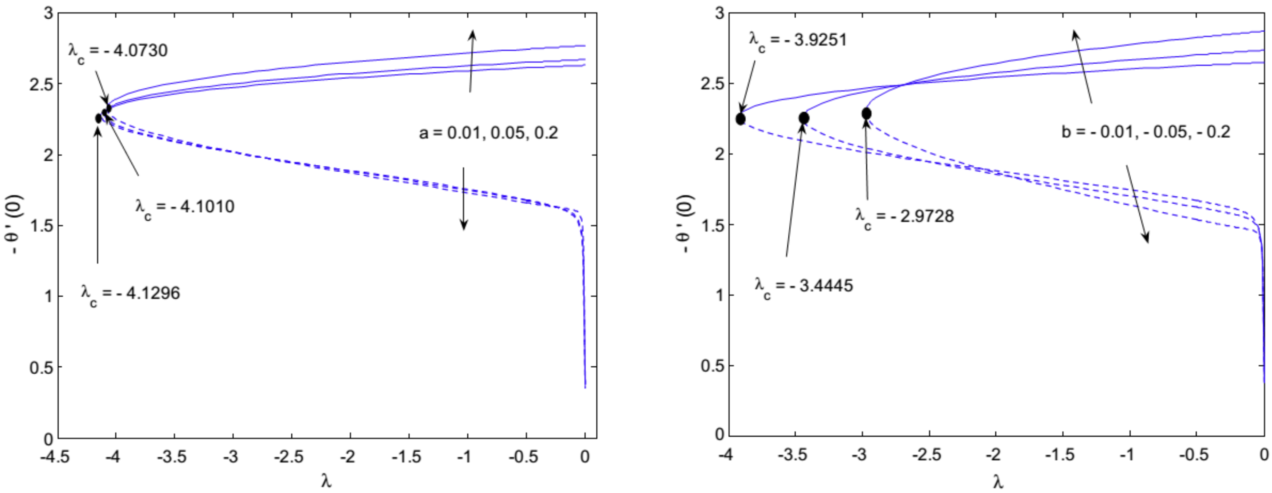
$$
\begin{aligned}
u_{\text{slip}}(x)&=\frac{2}{3}\left(\frac{3-\varepsilon l^2}{\varepsilon}-\frac{3}{2}\frac{1-l^2}{K_n}\right)\delta\frac{\partial u}{\partial y}-\frac{1}{4}\left[l^4+\frac{2}{K_n^2}(1-l^2)\right]\delta^2\frac{\partial^2 u}{\partial y^2}\\
&=A\frac{\partial u}{\partial y}+B\frac{\partial^2 u}{\partial y^2}
\end{aligned}
$$

Choosing appropriate similarity variables, the partial differential equations are transformed into ordinary (similarity) differential equations, which are then solved numerically using the function bvp4c from Matlab for different values of the governing parameters.

$$
\begin{aligned}
&\frac{\partial^3 f}{\partial\eta^3}+f\frac{\partial^2 f}{\partial\eta^2}-\left(\frac{\partial f}{\partial\eta}\right)^2-\frac{\partial^2 f}{\partial\eta\partial\tau}+\lambda\theta=0\\
&\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2}+f\frac{\partial\theta}{\partial\eta}-\frac{\partial f}{\partial\eta}\theta-\frac{\partial\theta}{\partial\tau}=0
\end{aligned}
$$

$$
\begin{aligned}
&f(0,\tau)=s, \quad \frac{\partial f}{\partial\eta}(0,\tau)=\sigma+a\frac{\partial^2 f}{\partial\eta^2}(0,\tau)+b\frac{\partial^3 f}{\partial\eta^3}(0,\tau), \quad \theta(0,\tau)=1\\
&\frac{\partial f}{\partial\eta}(\eta,\tau)\to 0, \quad \theta(\eta,\tau)\to 0 \quad \text{as } \eta\to\infty
\end{aligned}
$$

The solutions of the ordinary (similarity) differential equations have two branches, upper and lower branch solutions, in a certain range of the suction and mixed convection parameters. A stability analysis has been performed to show that the upper branch solutions are stable and physically realizable, while the lower branch solutions are not stable and, therefore, not physically possible. The effects of the two mass suction and mixed convection parameters on the reduced skin friction coefficient, heat transfer from the surface of the sheet, dimensionless velocity and temperature distributions are presented graphically and discussed. These results clearly show that the second order slip flow model is necessary to predict the flow characteristics accurately.

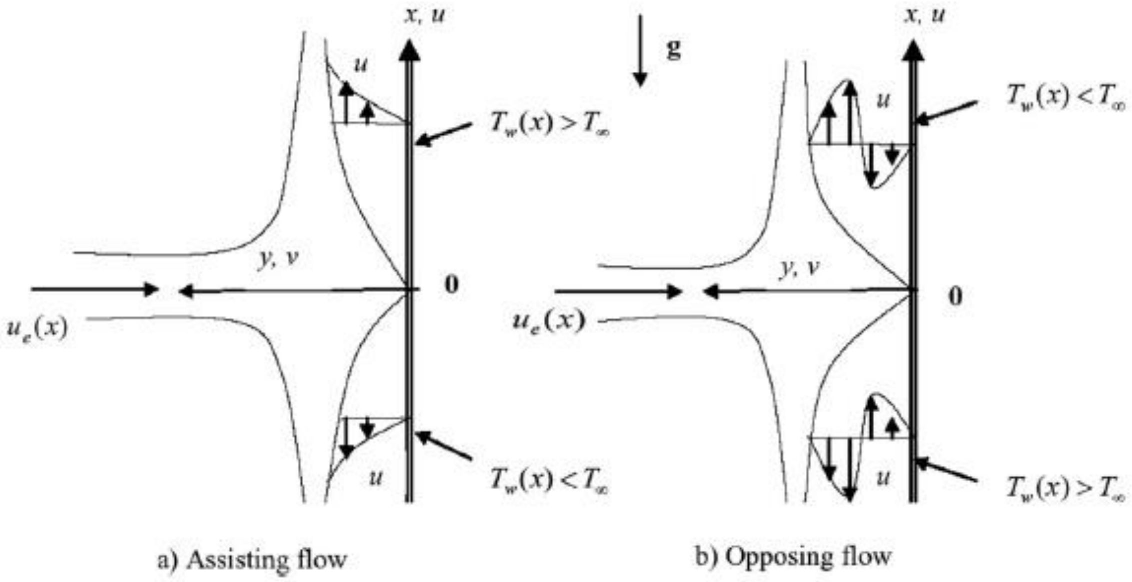


Variation of the domain of the dual solutions in respect with parameters a and b.

σ	λ	Upper branch solution	Lower branch solution
		γ	γ
1	−0.5	0.7662	−0.5803
	−0.3	1.1298	−0.6168
	−0.1	1.2958	−0.5943
−1	−0.5	0.2035	−0.1934
	−0.3	0.9329	−0.5200
	−0.1	1.0484	−0.6265

Eigenvalues for negative values of the parameter λ.

Paper 6. Mixed convection stagnation point flow past a vertical flat plate with a second order slip

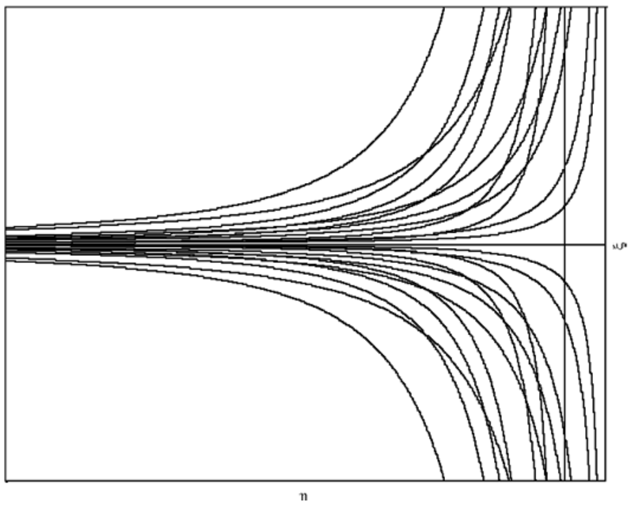


An exact similarity solution of the steady mixed convection flow of a viscous and incompressible fluid in the vicinity of two-dimensional stagnation-point with a second-order slip condition has been investigated. Using appropriate similarity variable, the Navier–Stokes equations coupled with the energy equation governing the flow and heat transfer are reduced to a system of nonlinear ordinary (similarity) equations, which are well-posed. These equations are solved numerically in the buoyancy assisting and opposing flow regions.

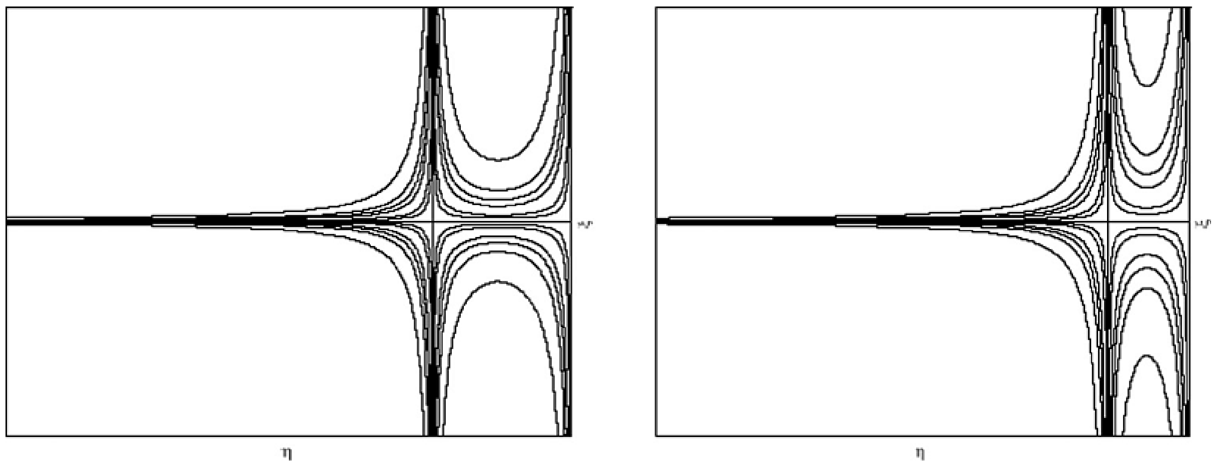
$$f''' + ff'' - f'^2 + 1 + \lambda\theta = 0 \quad , \qquad \frac{1}{\Pr}\theta'' + f\theta' - f'\theta = 0$$

$$f(0) = 0, \quad f'(0) = af''(0) + bf'''(0), \quad \theta(0) = 1$$
$$f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty$$

It is found that a reverse flow region develops in the buoyancy opposing flow case, and dual (upper and lower branch) solutions are found to exist in the case of opposing flow region for a certain range of the negative values of the mixed convection parameter. A stability analysis has been performed, which shows that the upper branch solutions are stable and physically realizable in practice, while the lower branch solutions are not stable and, therefore, not physically realizable in practice.



Streamlines for the first solution branch for for λ = -1.3 and λ=-1.5 when a = 1 and b = -1 .



Streamlines for the second solution branch for λ = -1.3 (left) and λ=-1.5 (right)when a = 1 and b = -1 .

Paper 7. Mixed convection boundary layer flow on a horizontal flat surface with a convective boundary condition

The steady mixed convection boundary layer flow on an upward facing horizontal surface heated convectively is considered. The problem is reduced to similarity form, a necessary requirement for which is that the outer flow and surface heat transfer coefficient are spatially dependent. The temperature at large distances from the plate is T_∞, while on the plate we have the following condition:

$$k\frac{\partial T}{\partial y} = -h_f(x)(T_f - T)$$

The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = g\beta(T - T_\infty)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$u = v = 0, \quad k\frac{\partial T}{\partial y} = -h_f(x)(T_f - T) \quad \text{on } y = 0$$

$$u \to U_\infty(x), \quad T \to T_\infty \quad \text{as } y \to \infty$$

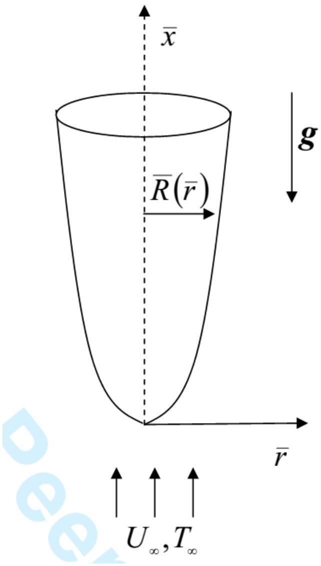
The above equations were solved numerically, but asymptotic solutions were also found for M=0 and M >> 1 and γ small or large where M is the miced convection parameter and γ is the Biot number given by

$$M = \frac{U_0}{(\nu(g\beta\Delta T)^2)^{1/5}}, \qquad \gamma = C_0\left(\frac{\nu^2}{g\beta\Delta T}\right)^{1/5}$$

In the table below numerical values for the dimensionless skin friction coefficient *f*'(0), pressure *P* and Nusselt number *Θ*(0) are given.

M	γ	$f''(0)$	$P(0)$	$\theta(0)$
0	0.1	0.443043	0.603379	0.267069
	1	0.821848	1.375053	0.747817
	10	0.958138	1.687192	0.965724
	1000	0.978190	1.734434	0.999642
1	0.01	0.643695	0.043371	0.026939
	0.1	0.782114	0.323480	0.209222
	1	1.092786	1.016773	0.708863
	10	1.226456	1.338960	0.959394
	1000	1.246939	1.389483	0.999575

Paper 8. Mixed convection boundary layer flow along vertical thin needles in nanofluids



The purpose of this paper is to theoretically study the problem of mixed convection boundary layer flow and heat transfer past a vertical needle with variable wall temperature using nanofluids.

$$\frac{\partial}{\partial x}(r u)+\frac{\partial}{\partial r}(r v)=0$$

$$u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial r}=u_e \frac{d u_e}{d x}+\frac{1}{(1-\phi)^{2.5}(1-\phi+\phi \rho_s / \rho_f)} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) + \frac{(1-\phi)+\phi(\rho \beta)_s /(\rho \beta)_f}{(1-\phi)+\phi \rho_s / \rho_f} \lambda T$$

$$u \frac{\partial T}{\partial x}+v \frac{\partial T}{\partial r}=\frac{1}{\Pr} \frac{k_{nf} / k_f}{(1-\phi)+\phi\,(\rho C_p)_s /(\rho C_p)_f} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

$$\begin{aligned} & v=0, \quad u=0, \quad T=T_w(x) \quad \text{at} \quad r=R(x) \\ & u=u_e(x), \quad T=0 \quad \text{as} \quad r \rightarrow \infty \end{aligned}$$

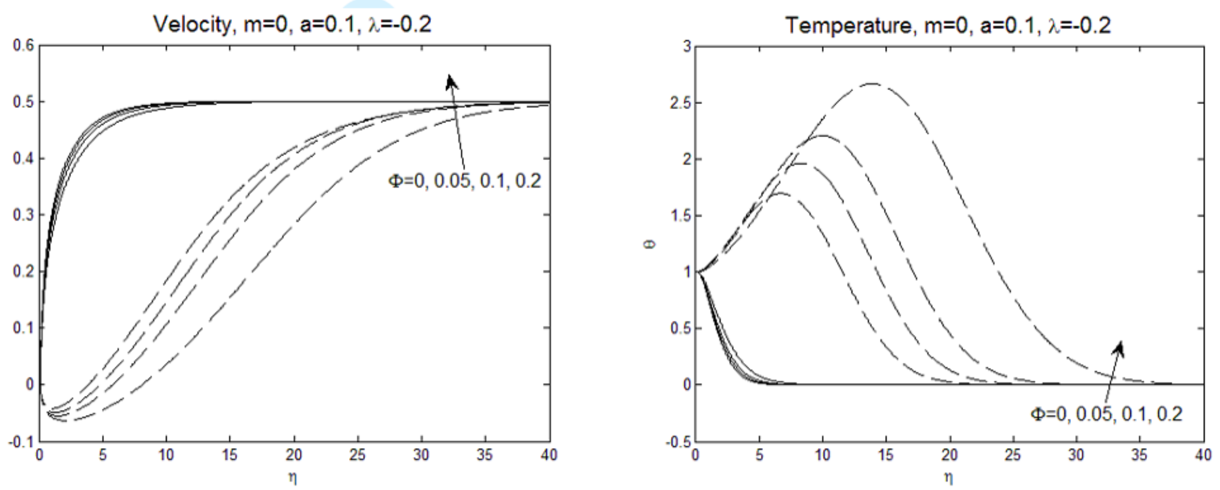
Using appropriate similar transformations the above equations reduce to ordinary differential equations which are solved using the function bv4c from MATLAB.

$$\frac{8}{(1-\phi)^{2.5}(1-\phi+\phi \rho_s / \rho_f)}(\eta f'')' + 4 f f''+m\left(1-4 f'^2\right) + \frac{(1-\phi)+\phi(\rho \beta)_s /(\rho \beta)_f}{(1-\phi)+\phi \rho_s / \rho_f} \lambda \theta=0$$

$$\frac{2}{\Pr} \frac{k_{nf} / k_f}{(1-\phi)+\phi\,(\rho C_p)_s /(\rho C_p)_f}(\eta \theta')'+f \theta'-(2 m-1) f' \theta=0$$

$$\begin{aligned} & f(a)=f'(a)=0, \quad \theta(a)=1 \\ & f'(\infty)=1 / 2, \quad \theta(\infty)=0 \end{aligned}$$

The relative tolerance was set to 1e-10. For the study of the stability the authors also used the bvp4c function in combination with chebfun package from Matlab. It is found that the solid volume fraction affects the fluid flow and heat transfer characteristics. The numerical results for a regular fluid and forced convection flow are compared with the corresponding results reported by Chen and Smith. The solutions exists up to a critical value of λ, beyond which the boundary layer separates from the surface and the solution based upon the boundary-layer approximations is not possible.

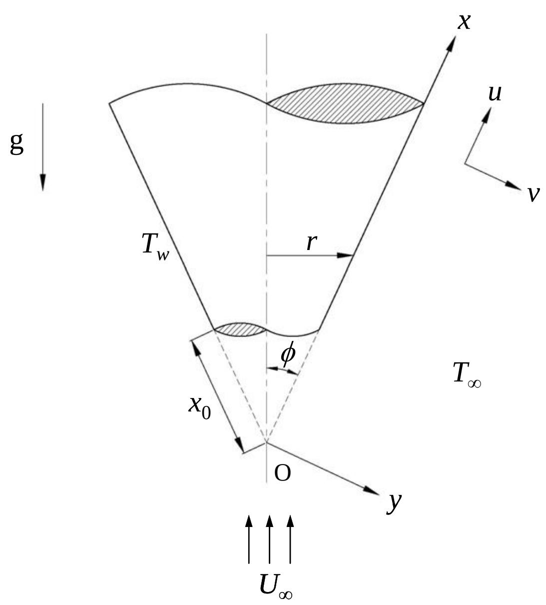


Stable and unstable velocity and temperature profiles for m = 0, λ = -0.2 and a = 0.1.

a	ϕ	γ	
		First solution	Second solution
0.1	0	0.578023	-0.218525
	0.05	0.706056	-0.429685
	0.1	0.785507	-0.607090
	0.2	0.881238	-0.910228
0.01	0	0.731600	-0.468000
	0.05	0.794625	-0.642135
	0.1	0.858591	-0.800051
	0.2	0.925993	-1.075026

Eigenvalues, γ, negative (unstable) and positive (stabile)

Paper 9. Mixed convection boundary layer flow from a vertical truncated cone in a nanofluid

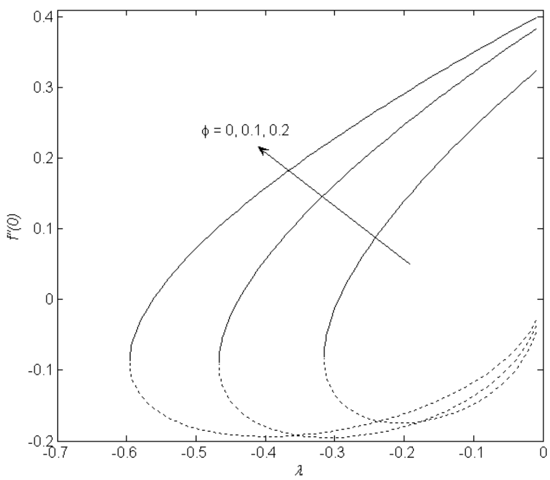


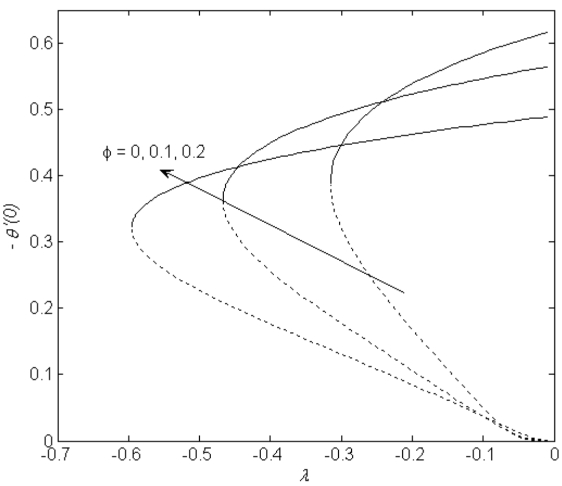
The purpose of this paper is to investigate the steady mixed convection boundary layer flow from a vertical frustum of a cone in water-based nanofluids. The mathematical model used for the nanofluid incorporates the particle volume fraction parameter, the effective viscosity and the effective thermal diffusivity. The entire regime of the mixed convection includes the mixed convection parameter, which is positive for the assisting flow (heated surface of the frustum cone) and negative for the opposing flow (cooled surface of the frustum cone), respectively.

It is found that dual solutions exist for the case of opposing flows. The range of the mixed convection parameter for which the solution exists increases in the presence of the nanofluids.

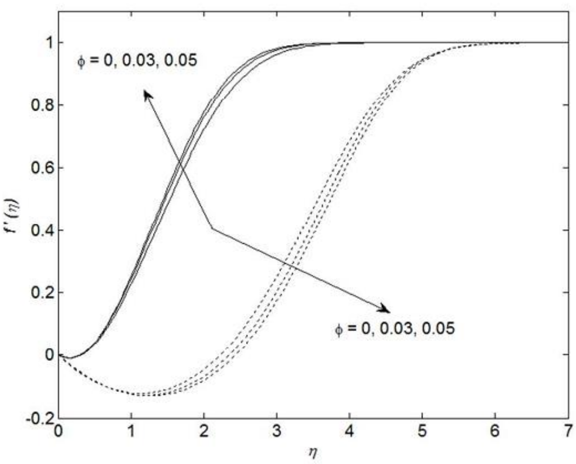
$$\frac{1}{(1-\varphi)^{2.5}\left[(1-\varphi)+\varphi\rho_s/\rho_f\right]}\frac{\partial^3 f}{\partial\eta^3}+\left(\frac{1}{2}+\frac{\xi}{1+\xi}\right)f\frac{\partial^2 f}{\partial\eta^2}$$
$$+\frac{(1-\varphi)+\phi(\rho\beta)_s/(\rho\beta)_f}{(1-\varphi)+\varphi\rho_s/\rho_f}\lambda\,\theta=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta}-\frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right)$$

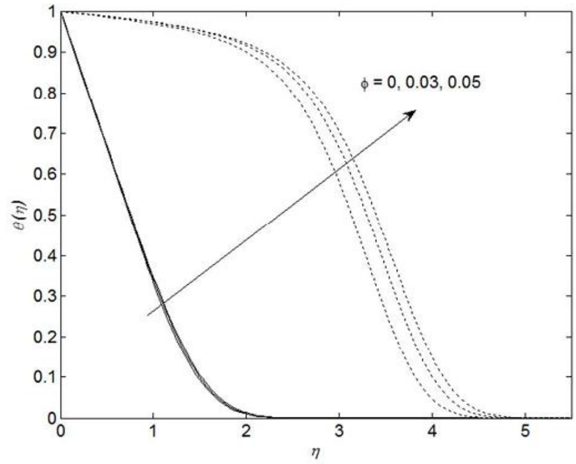
$$\frac{1}{\Pr}\frac{k_{nf}/k_f}{(1-\varphi)+\varphi(\rho C_p)_s/(\rho C_p)_f}\frac{\partial^2\theta}{\partial\eta^2}+\left(\frac{1}{2}+\frac{\xi}{1+\xi}\right)f\frac{\partial\theta}{\partial\eta}$$
$$=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right)$$





Variation of the skin friction coefficient $f''(0)$ and variation of the Nusselt number $\theta'(0)$ pentru $\xi = 0,\ \lambda < 0, \phi = 0, 0.1$ and 0.2.





Stable and unstable velocity and temperature profiles for ε >> 1 and λ = -0.2

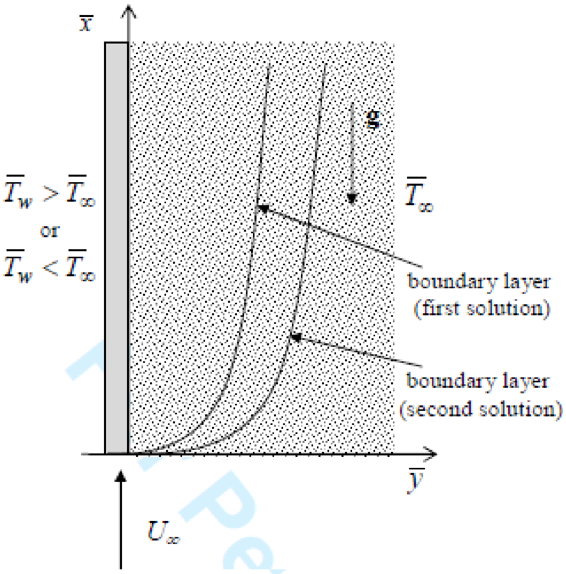
Paper 10. Mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium saturated by a nanofluid: Darcy-Ergun model

The purpose of this paper is to numerically solve the problem of steady mixed convection boundary layer flow past a vertical flat plate embedded in a fluid-saturated porous medium filled by a nanofluid. The non-Darcy equation model along with the mathematical nanofluid model proposed by Tiwari and Das (2007) has been used.

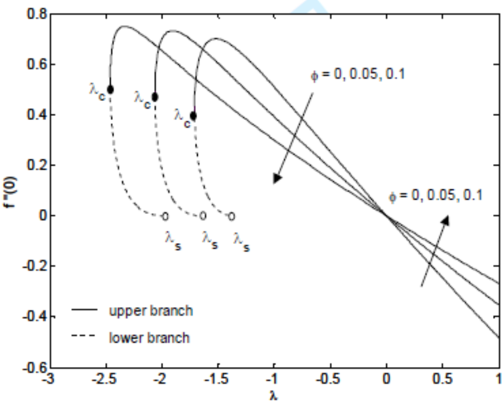
$$f'' + G(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)\left[(f')^2\right]' - \left[1-\phi+\phi(\rho\beta)_s/(\rho\beta)_f\right](1-\phi)^{2.5}\lambda\theta' = 0$$

$$\frac{k_{nf}/k_f}{1-\phi+\phi(\rho C_p)_s/(\rho C_p)_f}\theta'' + f\theta' = 0$$

$$f(0)=0,\quad \theta(0)=1$$
$$f'(\eta)\to 1,\quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty$$



Using appropriate similarity transformations, the basic partial differential equations are transformed into ordinary differential equations. These equations have been solved numerically for different values of the nanoparticle volume fraction, ϕ, the mixed convection λ and the non-Darcy G .parameters using the bvp4c function from Matlab. The results indicate that dual solutions exist for the opposing flow case ($\lambda < 0$).



Variation of $f''(0)$ with λ for ϕ = 0, 0.05, 1 and G = 0.3.

G	ϕ	λ_c	λ_s
0.3	0	-1.7123	-1.36
	0.05	-2.0651	-1.63
	0.1	-2.4659	-1.95

Critical values λ_c and λ_s for ϕ = 0, 0.05, 1 și G = 0.3.

Paper 11 . Stagnation point flow and heat transfer over a non-linearly moving flat plate in a parallel free stream with slip

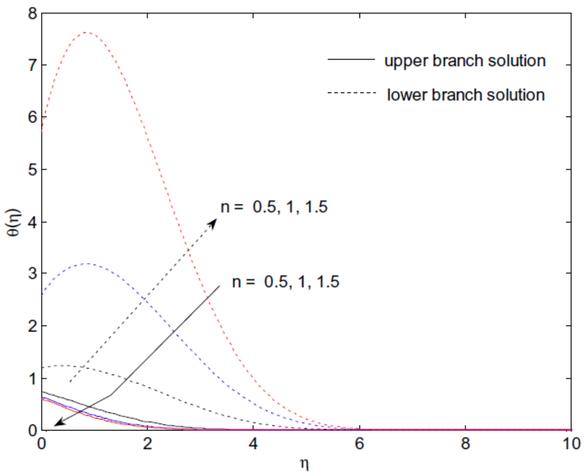
An analysis is presented for the steady boundary layer flow and heat transfer of a viscous and incompressible fluid in the stagnation point towards a non-linearly moving flat plate in a parallel free stream with a partial slip velocity. The governing differential equations are:

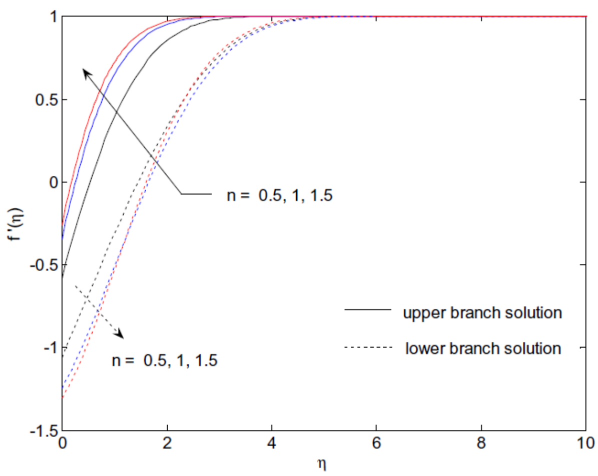
$$f''' + ff'' + \frac{2n}{n+1}(1 - f'^2) = 0$$

$$\frac{1}{\Pr}\theta'' + f\theta' - \frac{2p}{n+1}\, f'\theta = 0$$

$$f(0) = 0, \quad f'(0) = \lambda + \beta f''(0), \quad \theta(0) = 1 + \sigma\theta'(0), \quad f'(\infty) = 1, \quad \theta(\infty) = 0$$

The above mathematical model was solved numerically using the function bvp4c from Matlab for different values of the governing parameters. Dual (upper and lower branch) solutions are found to exist for certain parameters. Particular attention is given to deriving numerical results for the critical/turning points which determine the range of existence of the dual solutions. A stability analysis has been also performed to show that the upper branch solutions are stable and physically realizable, while the lower branch solutions are not stable and, therefore, not physically possible.



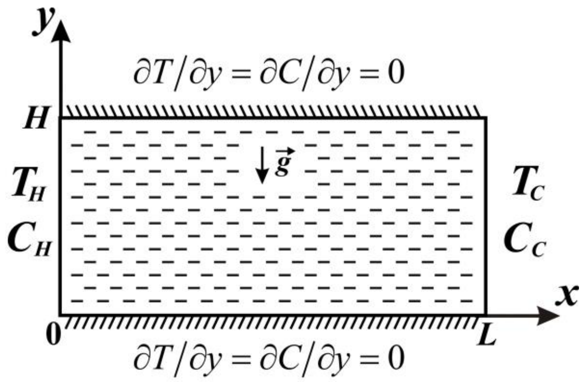


Moreover, the obtained results (values of the dimensionless Nusselt number) were compared in some particular cases with those existent in the open literature and a good agreement was observed.

	Pr				
	0.7	0.8	1	5	10
Present study	0.49586	0.52274	0.57046	1.04343	1.33879
Bejan [47]	0.496	0.523	0.570	1.043	1.344

Paper 12. Free Convection in Shallow and Slender Porous Cavities Filled by a Nanofluid Using Buongiorno's Model

A numerical study of the steady free convection flow in shallow and slender porous cavities filled by a nanofluid is presented. The nanofluid model takes into account the Brownian diffusion and the thermophoresis effects. The governing dimensional partial differential equations are transformed into a dimensionless form before being solved numerically using a finite difference method. Effort has been focused on the effects of four types of influential factors such as the aspect ratio, the Rayleigh and Lewis numbers, and the buoyancy-ratio parameter on the fluid flow and heat transfer characteristics.



Mathematical model:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} + Ra\left(\frac{\partial \theta}{\partial X} - N_r \frac{\partial \phi}{\partial X}\right) = 0$$

$$\frac{\partial \theta}{\partial \tau} + \frac{\partial \Psi}{\partial Y}\frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + N_b\left(\frac{\partial \phi}{\partial X}\frac{\partial \theta}{\partial X} + \frac{\partial \phi}{\partial Y}\frac{\partial \theta}{\partial Y}\right) + N_t\left[\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial X}\right)^2\right]$$

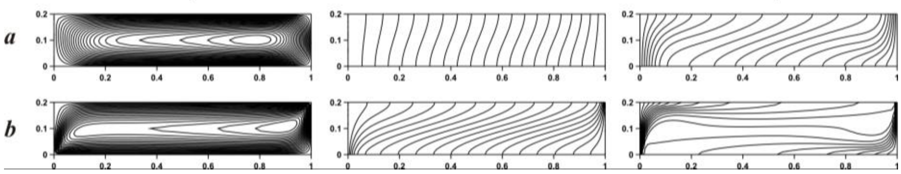
$$Le\left(\gamma \frac{\partial \phi}{\partial \tau} + \frac{\partial \Psi}{\partial Y}\frac{\partial \phi}{\partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \phi}{\partial Y}\right) = \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} + \frac{N_t}{N_b}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right)$$

$$\Psi = 0, \quad \frac{\partial \theta}{\partial Y} = \frac{\partial \phi}{\partial Y} = 0 \quad \text{on} \quad Y = 0, A$$

$$\Psi = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{on} \quad X = 0$$

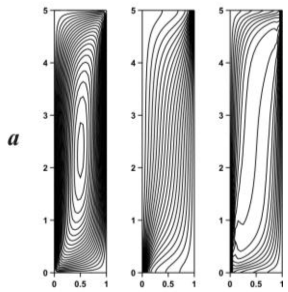
$$\Psi = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{on} \quad X = 1$$

The partial differential equations were solved numerically using a finite difference method using a non-uniform grid. Values of the Nusselt $\overline{Nu}$ and Sherwood $\overline{Sh}$ numbers and characteristics of the flow were obtained.



Stremlines, isotherms and concentrations lines for A = 0.2: Ra = 100 – a, Ra = 1000 – b

It has been found that the addition of nanoparticles can lead to an increase in the average Nusselt number with about 54%.



Stremlines, isotherms and concentrations lines for A = 5: Ra = 100 – a, Ra = 1000 – b

Paper 13. Mixed convection boundary-layer flow near the lower stagnation point of a horizontal circular cylinder with a second-order wall velocity condition and a constant surface heat flux

The mixed convection boundary-layer flow near the lower stagnation point of a horizontal circular cylinder with a second-order slip velocity model and a constant surface heat flux is investigated.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_e\frac{\mathrm{d}U_e}{\mathrm{d}x}+g\beta(T-T_\infty)\sin\alpha+\nu\frac{\partial^2 u}{\partial y^2},$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\nu}{\sigma}\frac{\partial^2 T}{\partial y^2},$$

$$v=0,\quad k\frac{\partial T}{\partial y}=-q_w \text{ on } y=0,\quad u\to\frac{U_0 x}{\ell},\quad T\to T_\infty \text{ as } y\to\infty\ \ (t>0),$$

A slip condition is imposed on cylinder surface

$$u=u_{\text{slip}}=M\frac{\partial u}{\partial y}+N\frac{\partial^2 u}{\partial y^2}\quad \text{on } y=0\ \ (t>0),$$

while the initial conditions are:

$$v=0,\quad u=0,\quad T=T_\infty \text{ at } t=0\ \ (0\leqslant y<\infty).$$

Using similar transformations the governing equations become:

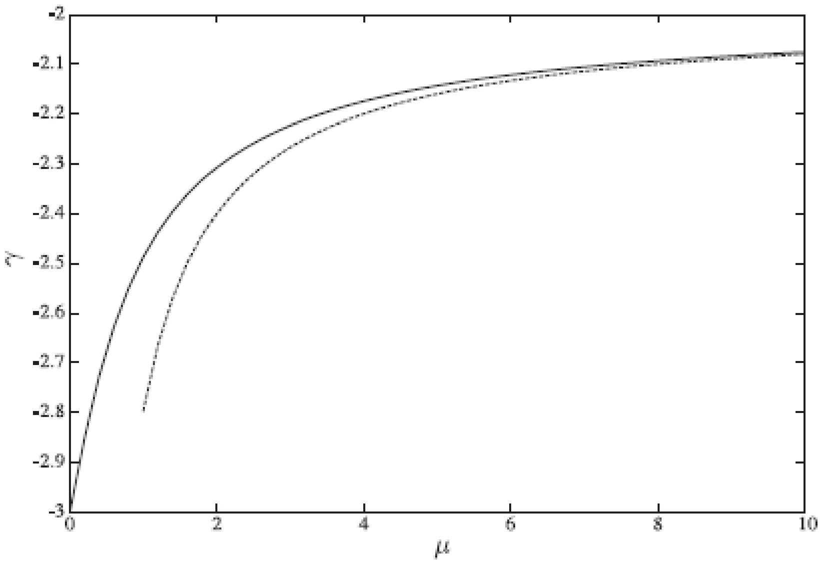
$$\frac{\partial^3 f}{\partial y^3}+\lambda\theta+f\frac{\partial^2 f}{\partial y^2}+1-\left(\frac{\partial f}{\partial y}\right)^2=\frac{\partial^2 f}{\partial y\partial t},$$

$$\frac{1}{\sigma}\frac{\partial^2\theta}{\partial y^2}+f\frac{\partial\theta}{\partial y}=\frac{\partial\theta}{\partial t},$$

$$f=0,\quad \frac{\partial f}{\partial y}=A\frac{\partial^2 f}{\partial y^2}+B\frac{\partial^3 f}{\partial y^3},\quad \frac{\partial\theta}{\partial y}=-1 \text{ on } y=0,\quad \frac{\partial f}{\partial y}\to 1,\quad \theta\to 0 \text{ as } y\to\infty,$$

$$\frac{\partial f}{\partial y}=0, \quad \theta=0 \text{ at } t=0 \ (0 \leqslant y < \infty).$$

The system of differential equations is solved numerically for different values of the governing parameters. These solutions can have two branches, an upper and a lower branch, over certain ranges of the mixed convection parameter. These numerical studies are complemented by the derivation of the asymptotic limits of the system parameters. A stability analysis is performed, showing that the upper branch is stable and physically realizable, whereas the lower branch solutions are unstable.

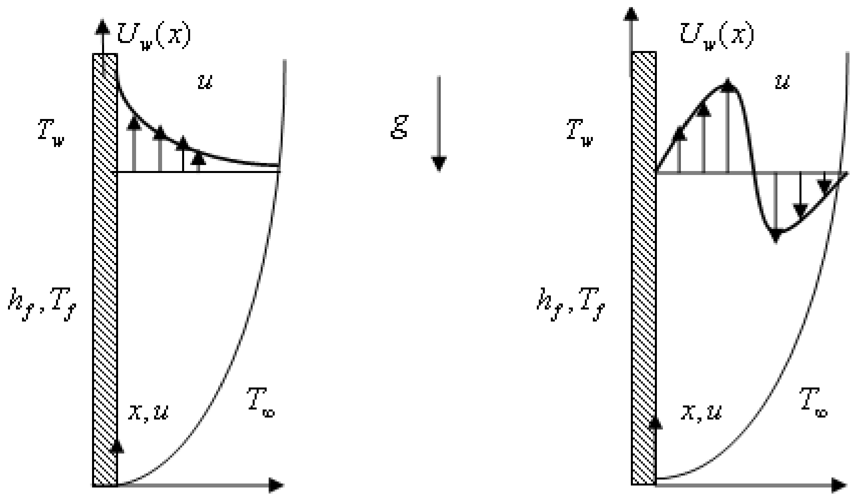


The eigenvalue γ in the large A, B limit plotted against μ, where B=μA. The asymptotic expression for large μ is shown by the broken line.

Paper 14. BOUNDARY LAYER FLOW OVER A MOVING VERTICAL FLAT PLATE WITH CONVECTIVE THERMAL BOUNDARY CONDITION

This paper studies the steady boundary layer flow over an impermeable moving vertical flat plate with convective boundary condition at the left side of the flat plate.

The governing partial differential equations are transformed into a system of ordinary (similarity) differential equations by using corresponding similarity variables. These equations were then solved numerically using the function bvp4c from Matlab for different values of the Rayleigh number Ra, the convective heat transfer parameter and the Prandtl number Pr. The paper demonstrates that a similarity solution is possible if the convective boundary condition heat transfer h_f associated with the hot or cooled fluid on the left side of the flat plate proportional to x−1/4.



The dimensionless governing equations are given by

$$f''' + \frac{3}{4} f f'' - \frac{1}{2} f'^2 + Ra\theta = 0$$

$$\frac{1}{Pr}\theta'' + \frac{3}{4} f\theta' = 0$$

$$f(0) = 0,\ f'(0) = \sigma,\ \theta'(0) = -\gamma[1 - \theta(0)]$$

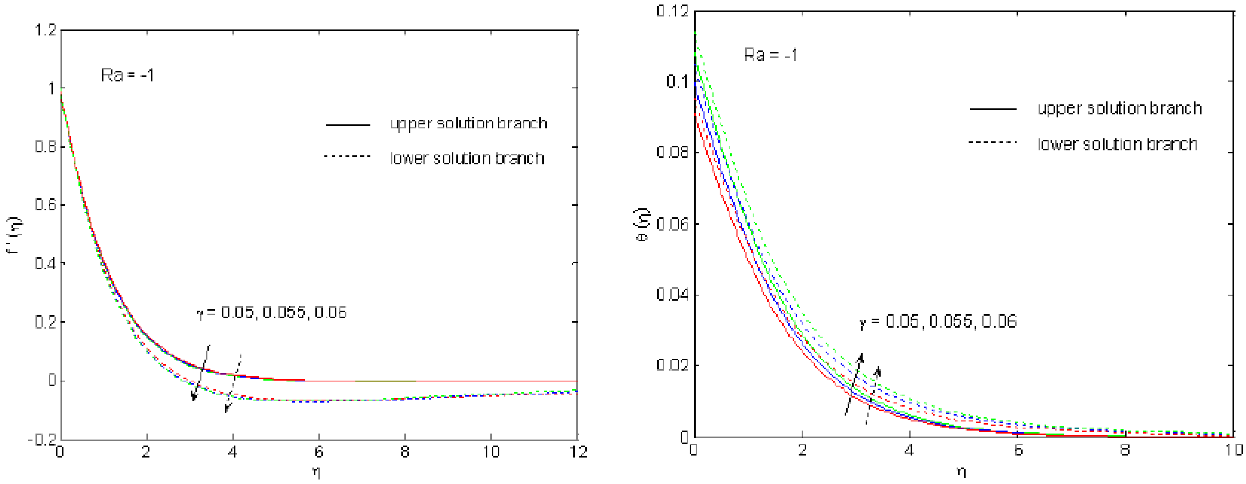
$$f'(\eta) \to 0,\ \theta(\eta) = 0 \text{ as } \eta \to \infty$$

The following conclusions can be made:

- Multiple (dual) solutions exist for both the cases of assisting (Ra > 0) and opposing (Ra < 0) cases.
- The stability analysis has revealed that the upper branch solutions are stable and physically realizable, while the lower branch solutions are unstable and, therefore, not physically realizable.
- It is found that there are critical values $Ra_c < 0$ of Ra < 0, with the values of $|Ra_c|$ decreasing with increasing γ for f''(0) and increasing with γ for $-\theta'(0)$, respectively.
- The reduced skin friction f''(0) decreases whilst the rate of heat transfer increases with the increasing of γ when the flow is opposing (Ra < 0).
- The velocity and temperature profiles are also affected by the governing parameters Ra and γ

Ra	γ	Upper branch	Lower branch
-0.5	0.05	0.2142	-0.0498
	0.1	0.1835	-0.0450
-1	0.05	0.2958	-0.0296
	0.1	0.2347	-0.0168

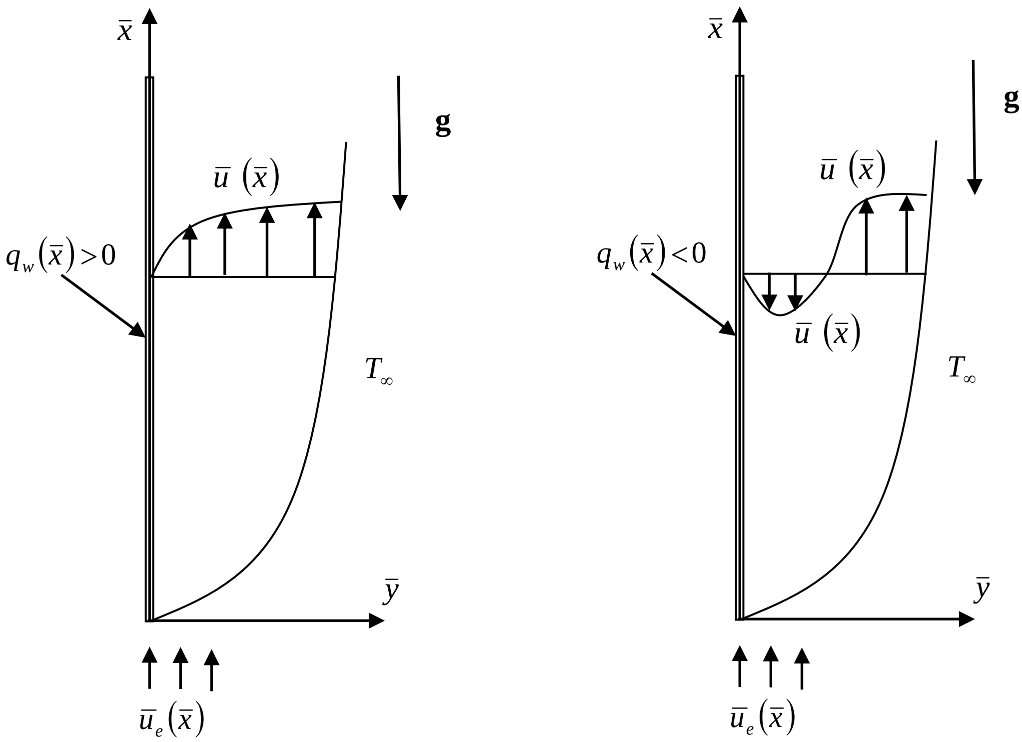
Smallest eigenvalues for several values of γ when Ra = −0.5 and −1 (opposing flow) with Pr = 1.



Stable (upper branch) and unstable (lower branch) velocity and temperature profiles

Paper 15. Mixed convection boundary layer flow past a vertical flat plate in a nanofluid: case of prescribed wall heat flux

An analysis is carried out to investigate the steady mixed convection boundary layer flow of a water based nanofluid past a vertical semi-infinite flat plate. Using an appropriate similarity transformation, the governing partial differential equations are transformed into coupled, nonlinear ordinary (similar) differential equations:



(assisting flow)

(opposing flow)

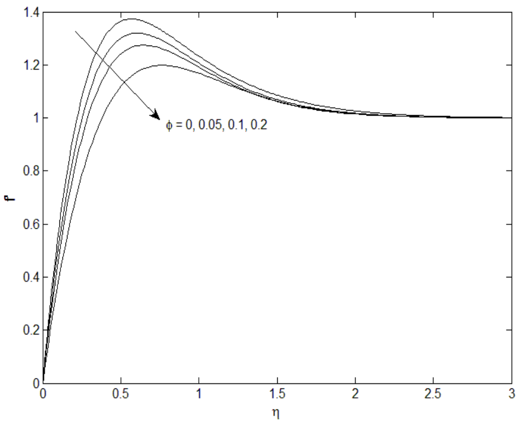
$$\frac{1}{(1-\phi)^{2.5}(1-\phi+\phi\rho_s/\rho_f)}f'''+\frac{m+1}{2}f\,f''+m(1-f'^2)$$
$$+\frac{(1-\phi)\;+\phi(\rho_s/\rho_f)(\beta_s/\beta_f)}{(1-\phi)\;+\phi\;(\rho_s/\rho_f)}\lambda\,\theta=0$$
$$\frac{1}{\Pr}\frac{k_{\mathrm{nf}}/k_{\mathrm{f}}}{(1-\phi)+\phi(\rho C_p)_s/(\rho C_p)_f}\theta''+\frac{m+1}{2}f\,\theta'+(1-2m)f'\theta=0$$
$$f(0)=f'(0)=0,\quad \frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}}\theta'(0)=-1$$
$$f'(\eta)\to 1,\quad \theta(\eta)\to 0\quad \text{as}\quad \eta\to\infty$$

where m is a positive paramete.

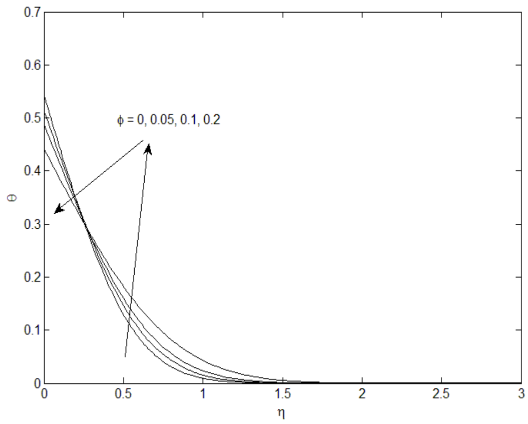
Values of $f''(0)$ and $\theta(0)$ for $m=1$ and $m=3/5$, $\Pr=1$ and $\Pr=6.2$, and for several values of ϕ when $\lambda>>1$ (free convection limit)

$\Pr$	m	ϕ	$f''(0)$	$\theta(0)$
1	1	0	1.009659 (1.00969)	1.513087 (1.51472)
	0.6	0	1.374060 (1.37438)	1.870563 (1.87284)
6.2	1	0	0.5168	0.9674
		0.05	0.4999	1.0384
		0.1	0.4813	1.1131
		0.2	0.4396	1.2772
	0.6	0	0.699	1.1809
		0.05	0.6763	1.2699
		0.1	0.6515	1.3632
		0.2	0.5956	1.568

(   ) Merkin and Mahmood [8]



a)



b)

a) Velocity $f'(\eta)$ and b) temperature $\theta(\eta)$ profiles for $\lambda=50$ and several values of ϕ when $m=1$.

Paper 16. Steady-state free convection in right-angle porous trapezoidal cavity filled by a nanofluid: Buongiorno's mathematical model

A numerical study of the steady free convection in a right-angle porous trapezoidal cavity filled by a nanofluid using Buongiorno's mathematical model has been performed. The analysis has been done for different values of the governing parameters *Nb*, *Nt*, *Nr*, *Le*, *A* and Rayleigh number Ra in the range $50 \le Ra \le 1000$. The dimensionless governing equations are:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-Ra\frac{\partial\theta}{\partial x}+Ra\cdot Nr\frac{\partial\phi}{\partial x}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+Nb\left(\frac{\partial\phi}{\partial x}\frac{\partial\theta}{\partial x}+\frac{\partial\phi}{\partial y}\frac{\partial\theta}{\partial y}\right)+Nt\left[\left(\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial y}\right)^2\right]$$

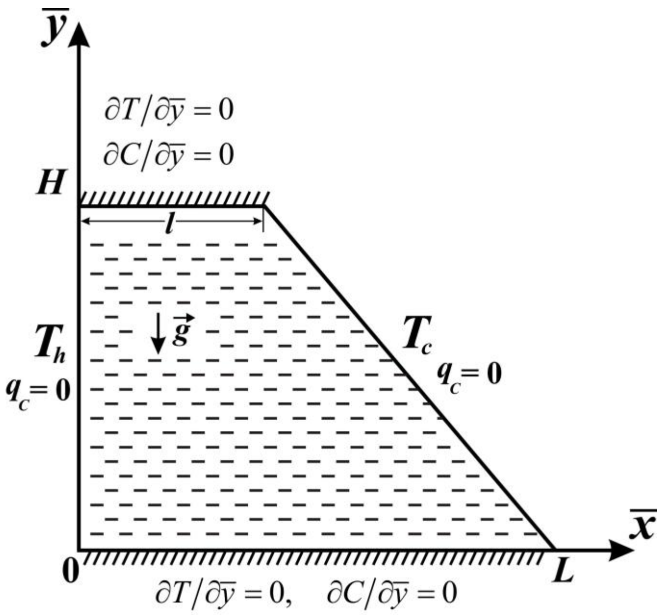
$$\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}=\frac{1}{Le}\left[\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right]+\frac{1}{Le}\frac{Nt}{Nb}\left[\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]$$

along with the boundary conditions:

$$\psi=0,\quad \theta=1,\quad \tilde{\mathbf{q}}_C=0\quad\left(\text{or}\quad Nb\frac{\partial\phi}{\partial x}+Nt\frac{\partial\theta}{\partial x}=0\right)\quad\text{on}\quad x=0$$

$$\psi=0,\quad \theta=0,\quad \tilde{\mathbf{q}}_C=0\quad\left(\text{or}\quad Nb\frac{\partial\phi}{\partial\mathbf{n}}+Nt\frac{\partial\theta}{\partial\mathbf{n}}=0\right)\quad\text{on}\quad x+y(1-A)=1$$

$$\psi=0,\quad \frac{\partial\theta}{\partial y}=0,\quad \frac{\partial\phi}{\partial y}=0\quad\text{on}\quad y=0\text{ and }y=1$$

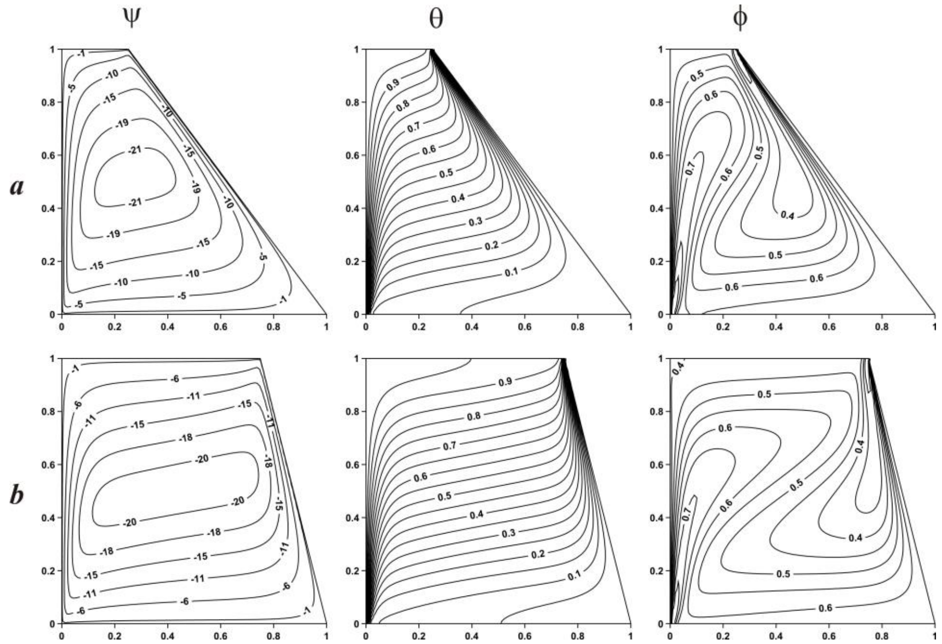


Finite difference method is used to solve governing equations. Central difference method is applied for discretization of equations. The solution of linear algebraic equations was performed using successive under relaxation (SUR) method. As convergence criteria, 10^{-10} is chosen for all dependent variables and value of 0.5 is taken for under-relaxation parameter. Next table compares the accuracy of the average Nusselt number in case of a square porous cavity for different values of the Rayleigh number at $Nr = Nb = Nt = 0$ (regular fluid) when the mass transfer is absent with some numerical solutions reported by different authors.

Authors	*Ra*			
	10	**100**	**1000**	**10000**
Walker and Homsy [18]	–	3.097	12.96	51.0
Bejan [19]	–	4.2	15.8	50.8
Beckerman et al. [20]	–	3.113	–	48.9
Gross et al. [21]	–	3.141	13.448	42.583
Manole and Lage [22]	–	3.118	13.637	48.117
Moya et al. [23]	1.065	2.801	–	–
Baytas and Pop [24]	1.079	3.16	14.06	48.33
Present results	1.071	3.104	13.839	49.253

Comparison of the average Nusselt number $\overline{Nu}$ of the hot wall.

Streamlines, isotherms and concentration lines were obtained for the involved parameters.



Paper 17. Modelarea matematică a fenomenelor convective în medii poroase

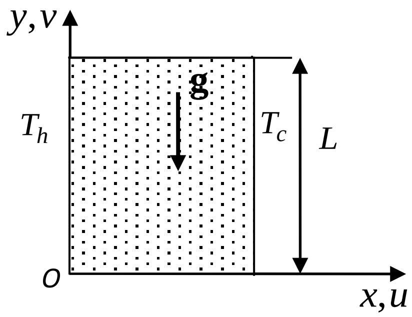
The book incorporate the main mathematical methods used in the research work related to this grant. It gives a short introduction in the theory of porous media, a detailed presentation of the porous media properties and mathematical models for transport phenomena in porous media. The momentum and energy equations are obtained using the averaging method. Moreover, numerical methods for ordinary differential equations (Cauchy and boundary values problems) and finite difference method for partial differential equations are discussed. Examples and practical applications are thoroughly presented (mathematical modeling, numerical solutions, results discussion).

Paper 18. . Metode de simulare Monte Carlo cu aplicații în economie.

The book describe in the first two chapters methods of type Monte Carlo for numerical integration. We intend for the last objective of this grant to approach problems dealing with the movement of nanoparticles using this kind of methods.

Lucrarea 19. Free convection in a porous cavity filled with nanofluids

The effect of different kind nanoparticles (cooper, alumina and titania) on free convection in a square cavity filled with a fluid-saturated porous medium have been investigated numerically. The top and bottom horizontal walls of cavity are considered adiabatic, while the vertical walls are kept at constant temperatures.. The governing equations are:



$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra\Gamma \frac{\partial \theta}{\partial X}$$

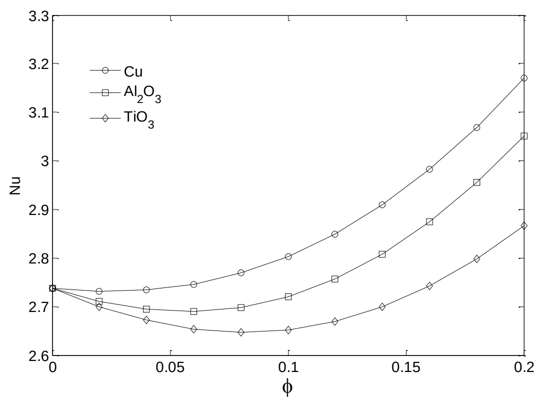
$$\frac{\partial \Psi}{\partial Y}\frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X}\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}$$

$$\Psi = 0, \theta = 1/2 \text{ on } X = 0$$

$$\Psi = 0, \theta = -1/2 \text{ on } X = 1$$

$$\Psi = 0, \frac{\partial \theta}{\partial Y} = 0 \text{ on } Y = 0, 1$$

In order to solve the above partial differential equations we used a central finite-difference discretization. The system of discretized equation was solved using a non-uniform grid near the walls. The equations have been solved using a Gauss-Seidel iterative scheme. The characteristics of fluid flow (streamlines and velocity profiles) and heat transfer (isotherms and Nusselt numbers) were obtained for different kind of nanoparticles. The effect of volume fraction of the nanoparticles is more present for Cu nano-particles and small Rayleigh numbers. When Al_2O_3 and TiO_3 nanoparticles are used the effects on fluid flow and heat transfer characteristics are similar. In the figure the variation of the Nusselt number Nu with ϕ is given for *Ra*=60.



Paper 20. Free convection heat transfer in a square cavity filled with a porous medium saturated by a water-based nanofluid

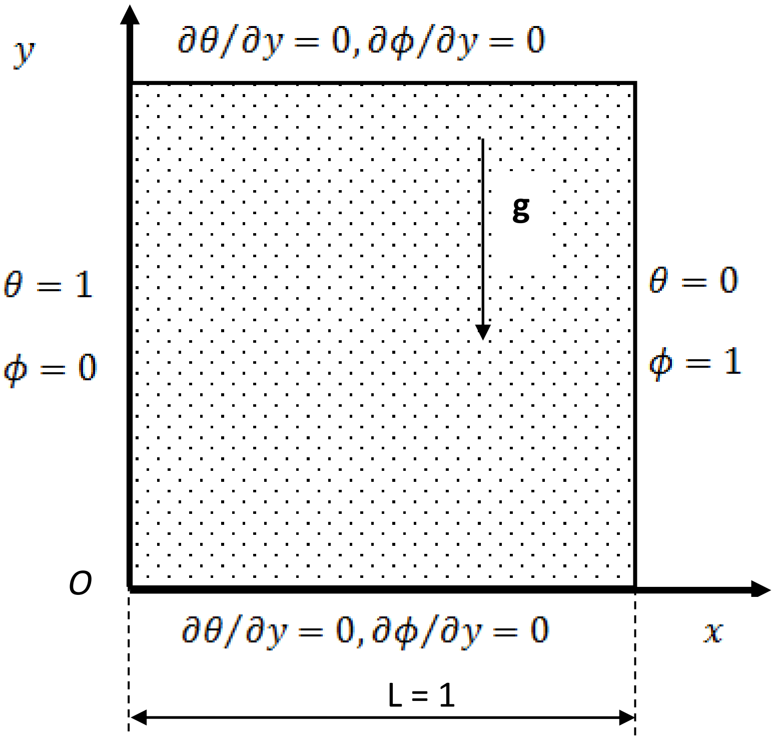
This paper presents a numerical study of natural convection heat transfer in a square cavity filled with a porous medium saturated by a nanofluid. The vertical walls are kept at the constant different temperatures and concentrations. The formulation of the problem is based on the Darcy-Boussinesq approximation, and following the nanofluid model proposed by Buongiorno:

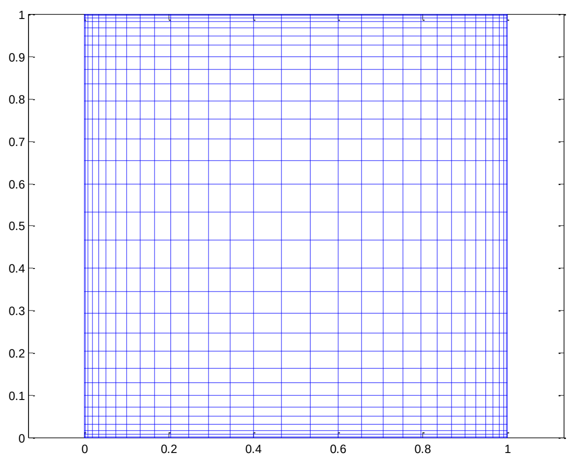
$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-Ra\frac{\partial\theta}{\partial x}+Nr\frac{\partial\phi}{\partial x}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+Nb\left(\frac{\partial\phi}{\partial x}\frac{\partial\theta}{\partial x}+\frac{\partial\phi}{\partial y}\frac{\partial\theta}{\partial y}\right)+Nt\left[\left(\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial y}\right)^2\right]$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}=\frac{1}{Le}\left[\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right]+\frac{1}{Le}\frac{Nt}{Nb}\left[\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]$$

$$\begin{aligned}&\psi=0,\quad \theta=1,\quad \phi=0\quad \text{on}\quad x=0\\&\psi=0,\quad \theta=0,\quad \phi=1\quad \text{on}\quad x=1\\&\psi=0,\quad \frac{\partial\theta}{\partial y}=0,\quad \frac{\partial\phi}{\partial y}=0\quad \text{on}\quad y=0,1\end{aligned}$$





Ra	Le	Nb, Nt	$\overline{Nu}\big\vert_{x=0}$	$\overline{Nu}\big\vert_{x=1}$	$\overline{Sh}\big\vert_{x=0}$	$\overline{Sh}\big\vert_{x=1}$
100	1	0.1	3.1821	3.0530	5.7634	5.6353
		0.2	3.2479	2.9899	5.8291	5.5720
		0.3	3.3146	2.9276	5.8955	5.5095
		0.4	3.3821	2.8663	5.9626	5.4478
	10	0.1	3.1672	3.0620	14.6687	14.5636
		0.2	3.2238	3.0141	14.7217	14.5121
		0.3	3.2836	2.9690	14.7753	14.4608
		0.4	3.3465	2.9264	14.8298	14.4098
1000	1	0.1	13.8770	13.3991	23.4802	23.0035
		0.2	14.1199	13.1642	23.7228	22.7681
		0.3	14.3653	12.9319	23.9680	22.5349
		0.4	14.6134	12.7021	24.2153	22.3042
	10	0.1	13.8253	13.4558	55.4209	55.0514
		0.2	14.0262	13.2866	55.6070	54.8674
		0.3	14.2381	13.1274	55.7944	54.6837
		0.4	14.4613	12.9779	55.9833	54.4999

Values of mean Nusselt and Sherwood numbers for $Nr = 0.4$.

In order to solve the governing partial differential equations we have used a central finite-difference discretization and the algebraic system of equation was solved using a Gauss-Seidel iterative scheme combined with the relaxation method It can be seen that the variation of the both $\overline{Nu}$ and $\overline{Sh}$ is relatively small with the variation of the *Nr*, while with the variation with the nanofluid parameters (*Nb* and *Nt*) is in the range of 4 to 7 percent being higher for lower values of *Ra* and *Le*. Comparing the values of $\overline{Nu}$ for the clear fluid and for nanofluid the addition of nanoparticles leads to an increase of the $\overline{Nu}$ with about 9% when $Ra = 100$ and with about 6% when $Ra = 1000$.

Paper 21. Mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with water at 4?C with a convective boundary condition: opposing flow case

The steady mixed convection boundary layer flow over a vertical impermeable surface embedded in a porous medium saturated with water close to its maximum density (at 4°C)

$$\frac{\rho - \rho_m}{\rho_m} = -\beta (T - T_m)^2$$

with convective boundary condition is theoretically studied.

$$-k \frac{\partial \bar{T}}{\partial \bar{y}} = h_f(\bar{x})(T_f - T_w)$$

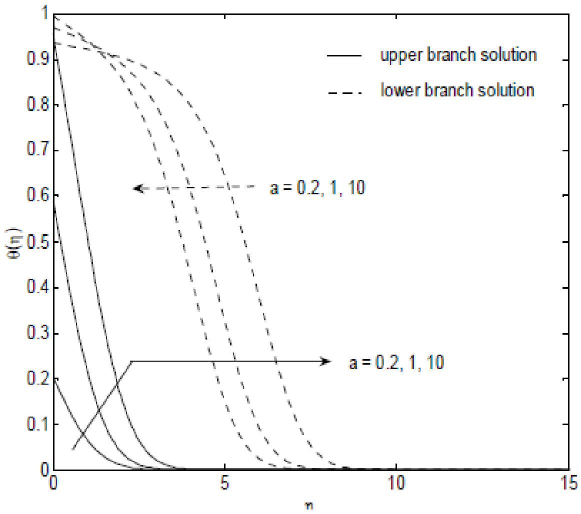
The governing partial differential equations are reduced to ordinary (similarity) differential equations:

$$f' = 1 - \lambda(\theta^2 + \delta\theta) \qquad f(0) = 0, \quad \theta'(0) = -a[1 - \theta(0)]$$

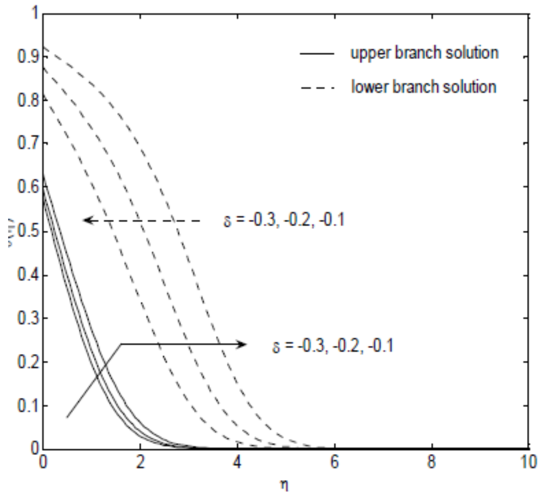
$$\theta'' + f\theta' = 0 \qquad , \qquad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty$$

where a =const., $\lambda = \dfrac{gK\beta(\Delta T)^2}{U_\infty \nu} = \dfrac{gK\beta(\Delta T)^2 L/(\alpha_m \nu)}{U_\infty L/\alpha_m} = \dfrac{Ra}{Pe}$ and $\delta = \dfrac{2(T_\infty - T_m)}{\Delta T}$.

These equations are solved for several values of the mixed convection parameter λ, convective parameter *a* and temperature parameter δ. The case of opposing flow is considered (λ ≥ 0). Dual solutions are found to exist, which are discussed in detail.

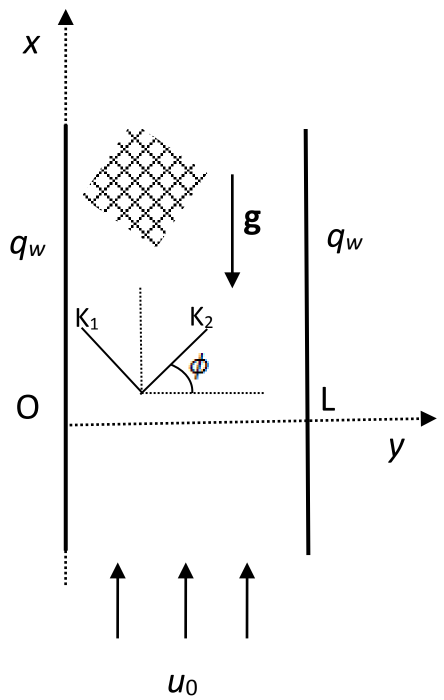


Temperature profiles for δ = -0.1 and λ = 1.7 .



Temperature profiles for a = 1 and λ = 3.

Paper 22. Fully developed assisting mixed convection flow through a vertical porous channel with an anisotropic permeability: Case of heat flux



The effect of anisotropy on the steady fully developed mixed convection flow in a vertical porous channel is analytically studied. The side walls of the channel are prescribed by a constant heat flux and the flow at the entrance is upward, so that natural convection aids the forced flow.:

$$\nabla \cdot \mathbf{v} = 0$$

$$\mathbf{v} = \frac{\overline{\overline{K}}}{\mu}\{-\nabla p + \mu\nabla^2\mathbf{v} + \rho[1 - \beta(T - T_0)]\mathbf{g}\}$$

$$\nabla \cdot (\mathbf{v}T) = \alpha_m \nabla^2 T,$$

Where the permeability is given by

$$\bar{\bar{K}} = \begin{bmatrix} K_1\cos^2\phi + K_2\sin^2\phi & (K_1 - K_2)\sin\phi\cos\phi \\ (K_1 - K_2)\sin\phi\cos\phi & K_2\cos^2\phi + K_1\sin^2\phi \end{bmatrix}.$$

Using the below dimensionless variables

$$X = \frac{\alpha_m x}{u_0 L^2},\quad Y = \frac{y}{L},\quad U(Y) = \frac{u}{u_0},\quad \theta(X,Y) = \frac{T - T_0}{q_w L/k},\quad P(X) = \frac{\alpha_m p}{\mu u_0^2 L}$$

we obtain the motion equations

$$\frac{d^2U}{dY^2} - \zeta^2 U + \lambda\theta + \gamma = 0,\quad U\frac{\partial\theta}{\partial X} = \frac{\partial^2\theta}{\partial Y^2}$$

With the boundary conditions:

$$U(0) = 0,\quad U(1) = 0,\quad \left.\frac{\partial\theta}{\partial Y}\right|_{Y=0} = -1,\quad \left.\frac{\partial\theta}{\partial Y}\right|_{Y=1} = 1,\quad \int_0^1 U dY = 1,\quad \int_0^1 \theta dY = 0$$

where

$$\zeta^2 = \sqrt{\frac{a}{Da}},\quad \gamma = -\frac{\partial P}{\partial X} = -\frac{dP}{dx},\quad \lambda = \frac{Gr}{\mathrm{Re}}.$$

Using the symbolic software Mathematica analytical solutions were obtained:

Case 1: $\Delta < 0$. U is given by

$U(Y) = C_1 e^{\alpha Y}\cos\beta Y + C_2 e^{\alpha Y}\sin\beta Y + C_3 e^{-\alpha Y}\cos\beta Y + C_4 e^{-\alpha Y}\sin\beta Y,$

where

$$C_1 + C_3 = 0$$

$$C_1 e^{\alpha}\cos\beta + C_2 e^{\alpha}\sin\beta + C_3 e^{-\alpha}\cos\beta + C_4 e^{-\alpha}\sin\beta = 0$$

$$\frac{2C_1\alpha}{\alpha^2+\beta^2} - \frac{2C_2\beta}{\alpha^2+\beta^2} - \frac{2C_3\alpha}{\alpha^2+\beta^2} - \frac{2C_4\beta}{\alpha^2+\beta^2} + C_5 = -1$$

$$\begin{aligned}(\alpha\cos\beta + \beta\sin\beta)e^{\alpha}C_1 + (\alpha\sin\beta - \beta\cos\beta)e^{\alpha}C_2 + (\beta\sin\beta - \alpha\cos\beta)e^{-\alpha}C_3 - (\alpha\sin\beta + \beta\cos\beta)e^{-\alpha}C_4 \\ + (\alpha^2+\beta^2)C_5 = \frac{\alpha^2+\beta^2}{2}\end{aligned}$$

$$\begin{aligned}&[((2\alpha^3 - 6\alpha\beta^2)\cos\beta + (6\beta\alpha^2 - 2\beta^3)\sin\beta)e^{\alpha} + 6\alpha\beta^2 - 2\alpha^3]C_1 \\ &+ [((2\beta^3 - 6\beta\alpha^2)\cos\beta + (2\alpha^3 - 6\alpha\beta^2)\sin\beta]e^{\alpha} + 6\beta\alpha^2 - 2\beta^3]C_2 \\ &+ [((6\alpha\beta^2 - 2\alpha^3)\cos\beta + (6\beta\alpha^2 - 2\beta^3)\sin\beta)e^{-\alpha} - 6\alpha\beta^2 + 2\alpha^3]C_3 \\ &+ [((2\beta^3 - 6\beta\alpha^2)\cos\beta + (6\beta\alpha^2 - 2\beta^3)\sin\beta)e^{-\alpha} + 6\beta\alpha^2 - 2\beta^3]C_4 \\ &+ (3\alpha^4\beta^2 + 3\alpha^2\beta^6 + \alpha^6 + \beta^6)C_5 + (6\alpha^4\beta^2 + 6\alpha^2\beta^6 + 2\alpha^6 + 2\beta^6)C_6 = 0\end{aligned}$$

Case 2: $\Delta = 0$. U is given by

$U(Y) = C_1 e^{rY} + C_2 Y e^{rY} + C_3 e^{-rY} + C_4 Y e^{-rY},$

where constants C_i can be obtained solving the system:

$$C_1 + C_3 = 0$$
$$C_1 e^r + C_2 e^r + C_3 e^{-r} + C_4 e^{-r} = 0$$
$$C_1 r - C_2 - C_3 r - C_4 + C_5 r^2 = -\frac{r^2}{2}$$
$$r e^r C_1 + (r-1) e^r C_2 - r e^{-r} C_3 - (r+1) e^{-r} C_4 + r^2 C_5 = \frac{r^2}{2}$$
$$2r(e^r - 1)C_1 + 2\big(3 + e^r(r-3)\big)C_2 - 2r(e^{-r} - 1)C_3 + 2\big(3 - e^{-r}(r+3)\big)C_4 + r^4 C_5 + 2r^4 C_6 = 0.$$

Case 3: $\Delta > 0$. *U* is given by

$$U(Y) = C_1 e^{\alpha Y} + C_2 e^{-\alpha Y} + C_3 e^{\beta Y} + C_4 e^{-\beta Y},$$

where constants C_i can be obtained solving the system:

$$C_1 + C_2 + C_3 + C_4 = 0$$
$$C_1 e^{\alpha} + C_2 e^{-\alpha} + C_3 e^{\beta} + C_4 e^{-\beta} = 0$$
$$\frac{2C_1}{\alpha} - \frac{2C_2}{\alpha} + \frac{2C_3}{\beta} - \frac{2C_4}{\beta} + C_5 = -1$$
$$\frac{2C_1 e^{\alpha}}{\alpha} - \frac{2C_2 e^{-\alpha}}{\alpha} + \frac{2C_3 e^{\beta}}{\beta} - \frac{2C_4 e^{-\beta}}{\beta} + C_5 = 1$$
$$2\beta^3(e^{\alpha} - 1)C_1 - 2\beta^3(e^{-\alpha} - 1)C_2 + 2\alpha^3(e^{\beta} - 1)C_3 - 2\alpha^3(e^{-\beta} - 1)C_4 + \alpha^3\beta^3 C_5 + 2\alpha^3\beta^3 C_6 = 0$$

Director of the grant,

Conf. Dr. Grosan Teodor

Grosan Teodor