GENERALIZED QUASI-CONTRACTIONS ON WEAK ORTHOGONAL METRIC SPACES

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Abstract. In this sequel, we introduce and study the concept of the weak orthogonal metric spaces as a generalization of the orthogonal metric spaces. Besides, we define and study the generalized quasi-contractions on such spaces and illustrate several non-trivial examples to endorse our obtained results. Among other things, as corollaries we obtain the main results of some of the pioneering articles existing in the literature. Finally, we answer the open question posed by Gordji et al. [On orthogonal sets and Banach fixed point theorem, Fixed Point Theory, 18(2):569-578, 2017].
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