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APPLICATIONS OF FIBRE CONTRACTION PRINCIPLE TO SOME CLASSES OF FUNCTIONAL INTEGRAL EQUATIONS

VERONICA ILEA*, DIANA OTROCOL**, IOAN A. RUS***
AND MARCEL-ADRIAN ŞERBAN****

*Babeş-Bolyai University, Faculty of Mathematics and Computer Science,
M. Kogălniceanu St. 1, RO-400084 Cluj-Napoca, Romania
E-mail: vdarzu@math.ubbcluj.ro

**Technical University of Cluj-Napoca,
Memorandumului St. 28, 400114, Cluj-Napoca, Romania,
and
Tiberiu Popoviciu Institute of Numerical Analysis,
Romanian Academy, P.O.Box. 68-1, 400110, Cluj-Napoca, Romania
E-mail: dotrocol@ictp.acad.ro

***Babeş-Bolyai University, Faculty of Mathematics and Computer Science,
M. Kogălniceanu St. 1, RO-400084 Cluj-Napoca, Romania
E-mail: iarus@math.ubbcluj.ro

****Babeş-Bolyai University, Faculty of Mathematics and Computer Science,
M. Kogălniceanu St. 1, RO-400084 Cluj-Napoca, Romania
E-mail: mserban@math.ubbcluj.ro

Abstract. Let $a < c < b$ real numbers, $(\mathbb{B}, |\cdot|)$ a (real or complex) Banach space, $H \in C([a,b]\times[a,c]\times\mathbb{B},\mathbb{B})$, $K \in C([a,b]^2 \times \mathbb{B},\mathbb{B})$, $g \in C([a,b],\mathbb{B})$, $A : C([a,c],\mathbb{B}) \to C([a,c],\mathbb{B})$ and $B : C([a,b],\mathbb{B}) \to C([a,b],\mathbb{B})$. In this paper we study the following functional integral equation,

$$x(t) = \int_a^c H(t,s,A(x)(s))ds + \int_a^t K(t,s,B(x)(s))ds + g(t),\ t \in [a,b].$$

By a new variant of fibre contraction principle (A. Petruşel, I.A. Rus, M.A. Şerban, Some variants of fibre contraction principle and applications: from existence to the convergence of successive approximations, Fixed Point Theory, 22 (2021), no. 2, 795-808) we give existence, uniqueness and convergence of successive approximations results for this equation. In the case of ordered Banach space $\mathbb{B}$, Gronwall-type and comparison-type results are also given.

Key Words and Phrases: Functional integral equation, Volterra operator, Picard operator, fibre contraction principle, Gronwall lemma, comparison lemma.
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