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ON MODIFIED $\mathcal{L}$-CONTRACTION VIA BINARY RELATION WITH AN APPLICATION

MD HASANUZZAMAN*, MOHAMMAD IMDAD** AND HAYEL N. SALEH***

*Department of Mathematics, Aligarh Muslim University, Aligarh, India
E-mail: md.hasanuzzaman1@gmail.com

**Department of Mathematics, Aligarh Muslim University, Aligarh, India
E-mail: mhimdad@gmail.com

***Department of Mathematics, Taiz University, Taiz, Yemen
E-mail: nasrhayel@gmail.com

Abstract. In this paper, we introduce the idea of $\mathcal{L}_{\mathcal{R}}$-contraction by employing an amorphous binary relation on $\mathcal{L}$-contraction in a metric space. We prove an existence and corresponding uniqueness fixed point results for $\mathcal{L}_{\mathcal{R}}$-contraction employing an S-transitive binary relation on metric spaces without completeness and also furnish an illustrative example to demonstrate the utility of our main results. Finally, we apply our newly obtained results to show the existence of a non-negative solution of the first-order ordinary differential equation.
Key Words and Phrases: Fixed points, $\mathcal{L}_{\mathcal{R}}$-contraction, binary relations, differential equation.
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