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A FIXED POINT DICHOTOMY

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Abstract. We give here a dichotomic fixed point result for a certain class of mappings defined in the closed unit ball of a Hilbert space. This dichotomy states that, for any of the mappings in this class, either it has a fixed point or its Lipschitz constant with respect to any renorming of ℓ_2 has to be strictly greater than 1.
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