

SPLIT EQUALITY FIXED POINT PROBLEMS AND COMMON NULL POINT PROBLEMS IN HILBERT SPACES

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Abstract. In this paper we introduce and study a new algorithm for finding a solution of the split equality fixed point problem for quasi-nonexpansive mappings and a common zero of a finite family of inverse strongly monotone mappings in Hilbert spaces. A numerical example to support our main theorem will be exhibited. Our results improve and generalize some recent results in the literature.

Key Words and Phrases: Split equality fixed point problems, inverse strongly monotone mappings, null point problem.

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REFERENCES

- [1] H.H. Bauschke, P.L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer-Verlag, 2011.
- [2] D.P. Bertsekas, E.M. Gafni, *Projection methods for variational inequalities with applications to the traffic assignment problem*, Math. Progr. Study, **17**(1982), 139-159.
- [3] C. Byrne, *Iterative oblique projection onto convex sets and the split feasibility problem*, Inverse Problems, **18**(2002), 441-453.
- [4] C. Byrne, *A unified treatment of some iterative algorithms in signal processing and image reconstruction*, Inverse Problems, **20**(2004), 103-120.
- [5] L.C. Ceng, Q.H. Ansari, J.C. Yao, *Some iterative method for finding fixed points and solving constrained convex minimization problems*, Numer. Algebra, Control Optim., **1**(3)(2011), 341-359.
- [6] Y. Censor, T. Bortfeld, B. Martin, A. Trofimov, *A unified approach for inversion problems in intensity-modulated radiation therapy*, Phys. Med. Biol., **51**(2006), 2353-2365.
- [7] Y. Censor, T. Elfving, *A multiprojection algorithms using Bragman projection in a product space*, Numerical Algorithms, **8**(1994), 221-239.
- [8] Y. Censor, A. Gibali, S. Reich, *Algorithms for the split variational inequality problem*, Numerical Algorithms, **59**(2012), 301-323.
- [9] M. Eslamian, *Strong convergence of split equality variational inequality and fixed point problem*, Riv. Mat. Univ. Parma, **8**(2)(2017), 225-246.

- [10] M. Eslamian, *Hybrid method for equilibrium problems and fixed point problems of finite families of nonexpansive semigroups*, Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales, Serie A. Matematicas, **107**(2013), 299-307.
- [11] M. Eslamian, G.Z. Eskandani, M. Raeisi, *Split common null point and common fixed point problems between Banach spaces and Hilbert spaces*, Mediterr. J. Math., **14**(3)(2017), 119.
- [12] A. Gibali, S. Reich, R. Zalas, *Iterative methods for solving variational inequalities in Euclidean space*, J. Fixed Point Theory Appl., **17**(2015), 775-811.
- [13] A. Gibali, S. Reich, R. Zalas, *Outer approximation methods for solving variational inequalities in Hilbert space*, Optimization, **66**(2017), 417-437.
- [14] K. Goebel, S. Reich, *Uniform Convexity, Hyperbolic Geometry, and Nonexpansive Mappings*, Marcel Dekker, New York and Basel, 1984.
- [15] H. Iiduka, W. Takahashi, *Strong convergence theorems for nonexpansive nonself-mappings and inverse-strongly-monotone mappings*, J. Convex Anal., **11**(2004), 69-79.
- [16] D. Kinderlehrer, G. Stampaccia, *An Iteration to Variational Inequalities and their Applications*, Academic Press, New York, 1990.
- [17] P.E. Mainge, *Strong convergence of projected subgradient methods for nonsmooth and non-strictly convex minimization*, Set-Valued Analysis, **16**(2008), 899-912.
- [18] E. Masad, S. Reich, *A note on the multiple-set split convex feasibility problem in Hilbert space*, J. Nonlinear Convex Anal., **8**(2007), 367-371.
- [19] A. Moudafi, *Alternating CQ-algorithm for convex feasibility and split fixed-point problems*, J. Nonlinear Convex Anal., **15**(2014), 809-818.
- [20] P.M. Pardalos, Th.M. Rassias, A.A. Khan, *Nonlinear Analysis and Variational Problems*, Springer, Berlin, 2010.
- [21] M. Raeisi, G.Z. Eskandani, M. Eslamian, *A general algorithm for multiple-sets split feasibility problem involving resolvents and Bregman mappings*, Optimization, **67**(2)(2018), 309-327.
- [22] M. Raeisi, G.Z. Eskandani, *A hybrid extragradient method for a general split equality problem involving resolvents and pseudomonotone bifunctions in Banach spaces*, Calcolo **56**, **43** (2019), <https://doi.org/10.1007/s10092-019-0341-4>.
- [23] G.Z. Eskandani, M. Raeisi, *Solving a general split equality problem without prior knowledge of operator norms in Banach spaces*, Results Math., **76**(2021), article no. 4, <https://doi.org/10.1007/s00025-020-01312-2>.
- [24] R.T. Rockafellar, R.J.-B. Wets, *Variational Analysis*, 2nd edition, Springer, New York, 2004.
- [25] W. Takahashi, M. Toyoda, *Weak convergence theorems for nonexpansive mappings and monotone mappings*, J. Optim. Theory Appl., **118**(2003), 417-428.
- [26] M. Tian, B.N. Jiang, *Weak convergence theorem for zero points of inverse strongly monotone mapping and fixed points of nonexpansive mapping in Hilbert space*, Optimization, **66**(2017), 1689-1698.
- [27] H.K. Xu, *Viscosity approximation methods for nonexpansive mappings*, J. Math Anal Appl., **298**(2004), 279-291.
- [28] H.K. Xu, *Iterative algorithms for nonlinear operators*, J. London Math. Soc., **66**(2002), 240-256.
- [29] I. Yamada, *The hybrid steepest descent method for the variational inequality problem over the intersection of fixed point sets of nonexpansive mappings*, In: Inherently Parallel Algorithms in Feasibility and Optimization and Their Applications, (D. Butnariu, Y. Censor, and S. Reich - Eds.) Amsterdam, Elsevier, 2001, 473-504.
- [30] J. Zhao, *Solving split equality fixed-point problem of quasi-nonexpansive mappings without prior knowledge of operators norms*, Optimization, **64**(2015), 2619-2630.

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