

A NOTE ON THE RATE OF CONVERGENCE OF VISCOSITY ITERATIONS

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Abstract. In [6], Moudafi introduced the so-called viscosity iterative method to approximate a fixed point of a nonexpansive mapping and proved the strong convergence of the generated sequence. Since then, several authors extended the convergence result in different settings and for mappings satisfying general metric conditions. Anyway, to the best of our knowledge and beside numerical simulations, little is known about the speed of convergence of the method itself. In this paper, we propose a step in this direction by giving an estimate for the rate of convergence of viscosity sequences generated by quasi-nonexpansive mappings in the setting of q -uniformly smooth Banach spaces.

Key Words and Phrases: Nonexpansive mapping, contractive mapping, iterative method, uniform smooth Banach space, duality map.

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