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Δ-CONVERGENCE OF CONVEX COMBINATIONS OF TWO MAPS ON p-UNIFORMLY CONVEX METRIC SPACES

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Abstract. In this paper, we first study some properties of the convex combination metric and the convex combination of two maps on a p-uniformly convex metric space. Also, we study the Δ-convergence of an iterative sequence for a convex combination of two maps on p-uniformly convex metric spaces.

Key Words and Phrases: Convex feasibility problem, p-uniformly convex metric spaces, convex combination of two maps, weighted average projection method, Δ-convergence, fixed point.
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