

A STOCHASTIC PRODUCTION PLANNING PROBLEM

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Abstract. Stochastic production planning problems were studied in several works; the model with one production good was discussed in [3]. The extension to several economic goods is not a trivial issue as one can see from the recent works [8], [9] and [13]. The following qualitative aspects of the problem are analyzed in [9]: the existence of a solution and its characterization through dynamic programming/Hamilton Jacobi Bellman (HJB) equation, as well as the verification (i.e., the solution of the HJB equation yields the optimal production of the goods). In this paper, we stylize the model of [8] and [9] in order to provide some quantitative answers to the problem. This is possible especially because we manage to solve the HJB equation in closed form. We point to a fixed point characterization of the optimal production rates. Among other results, we find that the optimal production rates adjusted for demand are the same across all the goods and they also turn to be independent of some model parameters. Moreover we show that production rates (adjusted for demand) are increasing in the aggregate number of goods produced, and they are also uniformly bounded. Numerical experiments show some patterns of the output.

Key Words and Phrases: Stochastic production problem, stochastic control, fixed point.

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