Fixed Point Theory, 23(2022), No. 1, 105-126
DOI: 10.24193/fpt-ro.2022.1.07
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

KRASNOSELSKII TYPE THEOREMS IN PRODUCT BANACH SPACES AND APPLICATIONS TO SYSTEMS OF NONLINEAR TRANSPORT EQUATIONS AND MIXED FRACTIONAL DIFFERENTIAL EQUATIONS

SANA HADJ AMOR*, RADU PRECUP** AND ABDELHAK TRAIKI*

*Department of Mathematics, LR 11 ES 35,
Higher School of Science and Technology,
University of Sousse, Tunisia
E-mail: sana.hadjamor@yahoo.fr, abdelhak.traikii@gmail.com

**Faculty of Mathematics and Computer Science
and
Institute of Advanced Studies in Science and Technology,
Babeş-Bolyai University, 400084 Cluj-Napoca, Romania
and
Tiberiu Popoviciu Institute of Numerical Analysis,
Romanian Academy, P.O. Box 68-1, 400110 Cluj-Napoca, Romania
E-mail: r.precup@math.ubbcluj.ro

Abstract. In this paper, we use a new technique for the treatment of systems based on the advantage of vector-valued norms and of the weak topology. We first present vector versions of the Leray-Schauder alternative and then some Krasnoselskii type fixed point theorems for a sum of two mappings. Applications are given to a system of nonlinear transport equations, and systems of mixed fractional differential equations.
Key Words and Phrases: Krasnoselskii fixed point theorem for a sum of operators, weak topology, generalized contraction, product Banach space, vector-valued norm, system of nonlinear transport equations, convergent to zero matrix, fractional integral.
2020 Mathematics Subject Classification: 47B38, 47H09, 47H08, 47H10.

References

[1] J. Appell, E. De Pascale, *Su alcuni parametri connessi con la misura di non compattezza di Hausdorff in spazi di funzioni misurabili*, Boll. Un. Mat. Ital., **B(6)**(1984), no. 3, 497-515.
[2] O. Arino, S. Gautier, J.P. Penot, *A fixed point theorem for sequentially continuous mappings with applications to ordinary differential equations*, Funkc. Ekvacial, **27**(1984), 273-279.
[3] C.S. Barroso, E.V. Teixeira, *A topological and geometric approach to fixed point results for sum of operators and applications*, Nonlinear Anal., **60**(2005), no. 4, 625-660.
[4] A. Ben Amar, I. Feki, A. Jerbi, *Leray-Shauder and Furi-Pera types fixed point theorems for the sum of two weakly sequentially continuous mappings and application to transport equation*, Afrika Mat., **25**(2014), 707-722.

[5] A. Ben Amar, A. Jeribi, M. Mnif, *On a generalization of the Schauder and Krasnoselskii fixed point theorems on Dunford-Pettis spaces and applications*, Math. Methods. Appl. Sci., **28**(2005), 1737-1756.
[6] A. Ben Amar, M. Mnif, *Leray-Schauder alternatives for weakly sequentially continuous mappings and application to transport equation*, Math. Methods Appl. Sci., **33**(2010), 80-90.
[7] T. Cardinali, R. Precup, P. Rubbioni, *Two abstract approaches in vectorial fixed point theory*, Quaest. Math., **41**(2018), 173-188.
[8] F.S. De Blasi, *On a property of the unit sphere in Banach spaces*, Bull. Math. Soc. Sci. Math. Roumanie, **21**(1977), 259-262.
[9] N. Dunford, J.T. Schwartz, *Linear Operators, Part I: General Theory*, Interscience Publishers, New York, 1958.
[10] R.E. Edwards, *Functional Analysis, Theory and Applications*, Holt, Reinhart and Winston, New York-Toronto-London, 1965.
[11] A. Guezane-Lakoud, S. Ramdane, *Existence of solutions for a system of mixed fractional differential equations. Journal of Taibah University for Science*, **12**(2018), no. 4, 421-426.
[12] I.M. James, *Topological Vector Spaces*, Springer, New York, 1987.
[13] M.A. Krasnosel'skii, *Two remarks on the method of successive approximations*, (Russian), Uspehi Mat. Nauk, **10**(1955), 123-127.
[14] K. Latrach, M.A. Taoudi, *Existence results for a generalized nonlinear Hammerstein equation on L_1 spaces*, Nonlinear Anal., **66**(2007), 2325-2333.
[15] K. Latrach, M.A. Taoudi, A. Zeghal, *Some fixed point theorems of the Schauder and the Krasnosel'skii type and application to nonlinear transport equations*, J. Diff. Equ., **221**(2006), 256-271.
[16] A.I. Perov, *On the Cauchy problem for a system of ordinary differential equations*, (Russian), Priblizhen. Metody Reshen. Differ. Uravn., **2**(1964), 115-134.
[17] I.-R. Petre, A. Petrusel, *Krasnoselskii's theorem in generalized Banach spaces and applications*, Electron. J. Qual. Theory Differ. Equ., **85**(2012), 20 pp.
[18] B.J. Pettis, *On integration in vector spaces*, Trans. Am. Math. Soc., **44**(1938), 277-304.
[19] R. Precup, *Methods in Nonlinear Integral Equations*, Kluwer, Dordrecht, 2002.
[20] R. Precup, *The role of matrices that are convergent to zero in the study of semilinear operator systems*, Mathematical and Computer Modelling, **49**(2009), no. 3-4, 703-708.
[21] H.H. Schaefer, *Neue existenzsä tze in der theorie nichtlinearer integragleichungen*, Sitzungsdh. Sächs. Akademie Wiss. Leipzig Matematisch-Naturwissenchaftliche, 1955, 101:7-40.
[22] A. Viorel, *Contributions to the study of nonlinear evolution equations*, Ph.D. Thesis, Babeş-Bolyai University, Cluj-Napoca, 2011.

Received: June 23, 2020; Accepted: September 19, 2020.