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ON EXISTENCE RESULTS IN FIXED SET THEORY AND APPLICATIONS TO SELF-SIMILARITY

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Abstract. In this manuscript, by removing the domain convexity hypothesis, the existence of fixed set results for the sum and the product of $(p+1)$-multi-valued operators $\sum_{i=1}^{p} A \cdot B_i$, acting on Banach algebras satisfying a sequential condition $(\mathcal{P})$ under weak topology is proved. In addition, by using a new definition of the multi-valued operator $\left(\frac{I}{A}\right)$, we obtain new fixed-set theorems for the operators of the form $\left(\frac{I}{A}\right)^{-1} \sum_{i=1}^{p} B_i$ under some suitable conditions on the operators $A, B_1, \ldots, B_p$. Applications to self-similarity theory are also given.

Key Words and Phrases: Banach algebra, weakly sequentially continuous, measure of weak noncompactness, fixed-set theory.
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References

[1] J. Appell, E. De Pascale, *Some parameters associated with the Hausdorff measure of noncompactness in spaces of measurable functions*, (Italien), Boll. Un. Mat. Ital. B. (6), **3**(1984), no. 2, 497-515.
[2] J. Banaś, M. Lecko, *Fixed points of the product of operators in Banach algebras*, Pan Amer. Math. J., **12**(2002), no. 2, 101-109.
[3] J. Banaś, O. Leszek, *On the equivalence of some concepts in the theory of Banach algebras*, Ann. Funct. Anal., **10**(2019), no. 2, 277-283.
[4] J. Banaś, L. Olszowy, *On a class of measures of noncompactness in Banach algebras and their application to nonlinear integral equations*, Z. Anal. Anwend., **28**(2009), no. 4, 475-498.
[5] J. Banaś, J. Rivero, *On measures of weak noncompactness*, Ann. Mat. Pura Appl., **151**(1988), no. 4, 213-224.

[6] J. Banaś, B. Rzepka, *Monotonic solutions of a quadratic integral equation of fractional order*, J. Math. Anal. Appl., **332**(2007), no. 2, 1371-1379.
[7] J. Banaś, K. Sadarangani, *Solutions of some functional-integral equations in Banach algebras*, Math. Comput. Modelling, **38**(2003), no. 3-4, 245-250.
[8] A. Ben Amar, I. Ben Hassine, M. Boumaiza, *Hybrid fixed-set results for multivalued mappings in Banach algebras under a weak topology setting*, Numer. Funct. Anal. Optim., **40**(2019), no. 6, 706-725.
[9] A. Ben Amar, M. Boumiza, D. O'Regan, *Hybrid fixed point theorems for multivalued mappings in Banach algebras under a weak topology setting*, J. Fixed Point Theory Appl., **18**(2016), no. 2, 327-350.
[10] A. Ben Amar, S. Chouayekh, A. Jeribi, *New fixed point theorems in Banach algebras under weak topology features and applications to nonlinear integral equations*, J. Funct. Anal., **259**(2010), no. 9, 2215-2237.
[11] A. Ben Amar, S. Chouayekh, A. Jeribi, *Fixed point theory in a new class of Banach algebras and application*, Afrika Mat., **24**(2013), no. 4, 705-724.
[12] D.W. Boyd, J.S.W. Wong, *On nonlinear contractions*, Proc. Amer. Math. Soc., **20**(1969), 458-464.
[13] J. Caballero, B. Lopez, K. Sadarangani, *Existence of nondecreasing and continuous solutions of an integral equations with linear modification of the argument*, Acta Math. Sin. (Engl. Ser.), **23**(2007), no. 9, 1719-1728.
[14] J.B. Conway, *A Course in Functional Analysis*, Graduate Texts in Mathematics, 96. Springer-Verlag, New York, 1990.
[15] F.S. De Blasi, *On a property of the unit sphere in a Banach space*, Bull. Math. Soc. Sci. Math. R.S. Roumanie (N.S.), **21**(69)(1977), no. 3-4, 259-262.
[16] B.C. Dhage, *Remarks on two fixed-point theorems involving the sum and the product of two operators*, Comput. Math. Appl., **46**(2003), no. 12, 1779-1785.
[17] B.C. Dhage, *A fixed point theorem in Banach algebras involving three operators with applications*, Kyungpook Math. J., **44**(2004), no. 1, 145-155.
[18] B.C. Dhage, *Fixed-point theorems for discontinuous multivalued operators on ordered spaces with applications*, Comput. Math. Appl., **51**(2006), no. 3-4, 589-604.
[19] B.C. Dhage, *On some nonlinear alternatives of Leray-Schauder type and functional integral equations*, Arch. Math. (Brno), **42**(2006), no. 1, 11-23.
[20] J.E. Hutchinson, *Fractals and self-similarity*, Indiana Univ. Math. J., **30**(1981), no. 5, 713-747.
[21] A. Jeribi, N. Kaddachi, B. Krichen, *Existence results for a coupled system of perturbed functional differential inclusions in Banach algebras*, Bull. Malays. Math. Sci. Soc., **41**(2018), no. 2, 893-918.
[22] A. Jeribi, B. Krichen, *Nonlinear Functional Analysis in Banach Spaces and Banach Algebras: Fixed Point Theory Under Weak Topology for Nonlinear Operators and Block Operator Matrices with Applications,* Monographs and Research Notes in Mathematics, CRC Press Taylor and Francis, 2015.
[23] K. Ben Amara, A. Jeribi, N. Kaddachi, *New fixed point theorems for countably condensing maps with an application to functional integral inclusions,* Math. Slovaca. **71**(2021), No. 6, 1–24.
[24] A. Fahem, A. Jeribi, N. Kaddachi, it Existence of Solutions for a System of Chandrasekhars Equations in Banach algebras under weak topology, Filomat **33**(2019); no:18; 5949-5957.
[25] A. Jeribi, N. Kaddachi, Z. Laouar, *Fixed point theorems for weakly asymptotically regular mappings in Banach spaces with an application,* to appear in Numerical Functional Analysis and Optimization.
[26] N. Kaddachi, A. Jeribi, B. Krichen, *Fixed Point Theorems of Block Operator Matrices On-Banach Algebras and an Application to Functional Integral Equations,* Math. Methods Appl. Sci. **36**(2013); 659-673.
[27] N. Kaddachi, *Existence results of ordinary differential inclusions in Banach algebra under weak topology,* J. Phys. Math. **9**(2018), DOI: 10.4172/2090-0902.1000285.
[28] N. Kaddachi, *Generalized form of fixed point theorems in Banach algebras under weak topology with an application,* Filomat **33**(2019), 4281–4296.

[29] N. Kaddachi, A. Jeribi, B. Krichen, *Fixed point theorems for multivalued operator matrices under weak topology with application,* Bull. Malays. Math. Sci. Soc. **43**(2020), no. 2, 1047–1067.
[30] R.W. Legget, *On certain nonlinear integral equations*, J. Math. Anal. Appl., **57**(1997), no. 2, 462-468.
[31] A. Majorana, S.A. Marano, *Continuous solutions of a nonlinear integral equation on an unbounded domain*, J. Integral Equations Appl., **6**(1994), no. 1, 119-128.
[32] B.B. Mandelbrot, *Fractals: Form, Chance, and Dimension*, translated from French, W.H. Freeman and Co., San Francisco, Califonia, 1977.
[33] G. Moţ, A. Petruşel, G. Petruşel, *Topics in Nonlinear Analysis and Applications to Mathematical Economics*, Casa Cărţii de Ştiinţă, Cluj-Napoca, 2007.
[34] E.A. Ok, *Fixed set theory for closed correpondences with applications to self similarity and games*, Nonlinear Anal. **56**(2004), no. 3, 309-330.
[35] E.A. Ok, *Fixed set theorems of Krasnoselskii type*, Proc. Amer. Math. Soc. **137**(2009), no. 2, 511-518.

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