

## GENERALIZED METRIC SPACES AND SOME RELATED FIXED POINT THEOREMS

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**Abstract.** In this paper, we study the existence of fixed point for relational endomorphisms. This class of mappings generalizes that of order and edge preserving mappings on posets and graphs respectively. As an application, we give a DeMarr-Type result for a family of Banach operator pairs in a binary relational system.

**Key Words and Phrases:** Binary relation, fixed point, generalized metric spaces, nonexpansive mapping, normal structure, relational endomorphisms.

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