RATE OF CONVERGENCE OF MODIFIED MANN ITERATION FOR ASYMPTOTICALLY NONEXPANSIVE MAPPINGS

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Abstract. Modified Mann iteration has been studied extensively for the approximation of fixed points of asymptotically nonexpansive mappings by many authors and known to be weakly convergent in the infinite-dimensional space. Our aim in this present paper is to provide a nonasymptotic $O(1/n)$ convergence rate result for a modified Mann iteration for asymptotically nonexpansive mappings in real Hilbert spaces.

Key Words and Phrases: Modified Mann iteration, asymptotically nonexpansive mappings, rate of convergence, Hilbert spaces.

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