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ON THE LOCAL CONVERGENCE OF HIGHER ORDER METHODS IN BANACH SPACES

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Abstract. We study the local convergence analysis of two higher-order methods using Hölder continuity condition on the first Fréchet derivative to solve nonlinear equations in Banach spaces. Hölder continuous first derivative is used to extend the applicability of the method on such problems for which Lipschitz condition fails. Also, this convergence analysis generalizes the local convergence analysis based on Lipschitz continuity condition. Our analysis provides the radius of convergence ball and error bounds along with the uniqueness of the solution. Numerical examples like Hammerstein integral equation and a system of nonlinear equations are solved to verify our theoretical results.
Key Words and Phrases: Banach space, local convergence, iterative methods, Hölder continuity condition.
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