A NEW PARALLEL ALGORITHM TO SOLVING A SYSTEM OF QUASI-VARIATIONAL INCLUSION PROBLEMS AND COMMON FIXED POINT PROBLEMS IN BANACH SPACES

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Abstract. In this paper, a new parallel algorithm for finding a common solution of a system of quasi-variational inclusion problems and a common fixed point of a finite family of nonexpansive mappings in a q-uniformly Banach space is introduced and analyzed. A strong convergence theorem of the proposed algorithm is established under some control conditions. As a consequence, we apply our main results to solve convex minimization problems, multiple sets variational inequality problems and multiple sets equilibrium problems. Some numerical experiments of image restoration problems are also given for supporting the main results.

Key Words and Phrases: Maximal monotone operator, inverse strong accretive operator, variational inclusion problem, Banach space, strong convergence.

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