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FIXED POINT THEOREMS IN THE STUDY OF OPERATOR EQUATIONS IN ORDERED BANACH SPACES AND THEIR APPLICATIONS

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Abstract. We use fixed point index properties and the general minorant principle (see Theorem 7.B in [12]) to prove new fixed point theorems for operators leaving invariant a cone in a Banach space. Main ideas of this work are inspired from the work in [11]. The results obtained are used to prove existence of at least one positive solution to a $\phi-$laplacian boundary value problem.
Key Words and Phrases: Cones, fixed point theory, positive solution, general minorant principle, boundary value problem.
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