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SOLUTION TO SECOND ORDER DIFFERENTIAL EQUATIONS VIA F_w-CONTRACTIONS

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Abstract. In this article, we introduce the notions of F-contractions and Hardy-Rogers type F-contractions via w-distances in the backdrop of an orthogonal metric space. After this, we prove some fixed point results concerning the said kind of contractions by taking a weaker version of completeness of the underlying space instead of completeness. Further, we employ the results to obtain some existence and uniqueness criteria of the solution(s) to a certain type of second order initial value and boundary value problems. Along with these, we illustrate some numerical examples to interpret our achieved fixed point results.
Key Words and Phrases: F-contractions, w-distances, orthogonal metric spaces, second order differential equations.
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