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ON INERTIAL TYPE ALGORITHMS WITH GENERALIZED CONTRACTION MAPPING FOR SOLVING MONOTONE VARIATIONAL INCLUSION PROBLEMS

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Abstract. In this article, we introduced two iterative processes consisting of an inertial term, forward-backward algorithm and generalized contraction for approximating the solution of monotone variational inclusion problem. The motivation for this work is to prove the strong convergence of inertial-type algorithms under some relaxed conditions because many of the existing results in this direction have only achieved weak convergence. We note that when the space is finite dimension, there is no disparity between weak and strong convergence, however this is not the case in infinite dimension. We provide some numerical examples to justify that inertial algorithms converge faster than non-inertial algorithms in terms of number of iterations and cpu time taken for the computation.
Key Words and Phrases: Accelerated algorithm, fixed point problem, inertial term, inverse strongly monotone, maximal monotone operators, zero problem.
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