EXISTENCE AND STABILITY OF COUPLED FIXED POINT SETS FOR MULTI-VALUED MAPPINGS

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Abstract. The existence of a coupled fixed point for a multi-valued mapping satisfying a certain admissibility condition is established. Requirement that the mapping be a contraction is met implicitly through a function of several variables. Further, we study the stability of coupled fixed point sets for such mappings. Several illustrative examples are presented.

Key Words and Phrases: Metric space, admissible maps, implicit relation, coupled fixed point, stability.

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