PSEUDOMONOTONE VARIATIONAL INEQUALITIES AND FIXED POINTS

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Abstract. We introduce two new iterative algorithms with line-search process for solving a variational inequality problem with pseudomonotone and Lipschitz continuous mapping and a common fixed-point problem of an asymptotically nonexpansive mapping and a strictly pseudocontractive mapping. The proposed algorithms are based on inertial subgradient extragradient method with line-search process, hybrid steepest-descent method, and viscosity approximation method. Under mild conditions, we prove strong convergence of the proposed algorithms in a real Hilbert space.

Key Words and Phrases: Inertial subgradient extragradient method, pseudomonotone variational inequality, nonexpansive mapping, strictly pseudocontractive mapping.

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