BOUNDARY VALUE PROBLEM FOR DIFFERENTIAL EQUATIONS WITH GENERALIZED HILFER-TYPE FRACTIONAL DERIVATIVE

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Abstract. In this paper, we establish the existence and uniqueness of solutions to boundary value problem for differential equations with generalized Hilfer type fractional derivative. The arguments are based upon the Banach contraction principle and Krasnoselskii's fixed point theorem. An example is included to show the applicability of our results.
Key Words and Phrases: Generalized Hilfer type fractional derivative, boundary value problem, existence, uniqueness, fixed point.
2020 Mathematics Subject Classification: 26A33, 34A08, 34B15, 47H10.

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