SURJECTIVITY RESULTS FOR NONLINEAR WEAKLY SEQUENTIALLY CONTINUOUS OPERATORS AND APPLICATIONS

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Abstract. We introduce new surjectivity results for a couple (T, S) where T and S are two weakly sequentially continuous operators and satisfy some homogeneity conditions. Also we present new operator quantities using the measure of weak noncompactness. Our results extend in a broad sense some theorems of Fučik, Nečas, Souček and Souček in the weak topology setting. In addition we present an application for generalized Hammerstein type integral equations.

Key Words and Phrases: Weakly sequentially continuous, a-homogeneous operator, measure of weak noncompactness.

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