EXISTENCE OF GROUP NONEXPANSIVE RETRACTIONS AND ERGODIC THEOREMS IN TOPOLOGICAL GROUPS

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Abstract. Suppose that G is a topological group and C a compact subset of G. In this paper we define group nonexpansive mappings and then we consider $\mathcal{S} = \{T_i : i \in I\}$ as a family of the group nonexpansive mappings on C. Also we study the existence of group nonexpansive retractions P_i from C onto $\mathrm{Fix}(\mathcal{S})$ such that $P_i T_i = T_i P_i = P_i$.

Key Words and Phrases: Fixed point, group nonexpansive mapping, topological group, retraction.

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