# POSITIVE SOLUTIONS FOR FRACTIONAL LAPLACIAN SYSTEM INVOLVING CONCAVE-CONVEX NONLINEARITIES AND SIGN-CHANGING WEIGHT FUNCTIONS

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**Abstract.** In this paper, we consider a fractional Laplacian system with both concave-convex nonlinearities and sign-changing weight functions in bounded domains. With the help of the Nehari manifold, we prove that the system has at least two positive solutions when the pair of the parameters $(\lambda, \mu)$ belongs to a certain subset of $\mathbb{R}^n$.

**Key Words and Phrases**: Fractional Laplacian, critical exponent, subcritical exponent, ground state solution, fixed point.

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