APPROXIMATING COMMON FIXED POINTS OF ASYMPTOTICALLY NONEXPANSIVE COSINE FAMILIES IN HILBERT SPACES BY ALGORITHMS WITH ERROR TERMS

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Abstract. In this paper the Mann type iterative scheme with error term to approximate a common fixed point of one-parameter asymptotically nonexpansive cosine family is investigated in Hilbert spaces. By using the theory of cosine families, some strong convergence theorems of the sequences generated by these schemes are established on closed convex subsets and compact convex subsets, respectively. As special cases, strong convergence results for nonexpansive cosine families are also obtained.

Key Words and Phrases: Asymptotically nonexpansive cosine family, common fixed point, Mann type iterative scheme with error term, strong convergence.

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