

FIXED POINT OF MULTIVALUED CONTRACTIONS BY ALTERING DISTANCES WITH APPLICATION TO NONCONVEX HAMMERSTEIN TYPE INTEGRAL INCLUSIONS

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Abstract. A new contraction condition for multivalued maps in metric spaces is introduced and then, based on this new condition, we prove two fixed point theorems for such contractions. The new condition uses the altering distance technique and a Pompeiu type metric on the family of nonempty and closed subsets of a metric space. Our results essentially compliments and generalizes some well known results. As application, we model a nonconvex Hammerstein type integral inclusion and prove an existence theorem for this problem.

Key Words and Phrases: Fixed point, metric space, Hausdorff metric, multivalued contraction, Hammerstein type integral inclusion.

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