BOUNDARY VALUE PROBLEMS FOR FRACTIONAL-ORDER DIFFERENTIAL INCLUSIONS IN BANACH SPACES WITH NONDENSELY DEFINED OPERATORS

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Abstract. We consider a nonlocal boundary value problem for a semilinear differential inclusion of a fractional order in a Banach space assuming that its linear part is a non-densely defined Hille-Yosida operator. We apply the theory of integrated semigroups, fractional calculus and the fixed point theory of condensing multivalued maps to obtain a general existence principle. An example of a concrete realization of this result is also given. Some important particular cases including a nonlocal Cauchy problem, periodic and anti-periodic boundary value problems are presented.
Key Words and Phrases: Fractional differential inclusion, boundary value problem, nonlocal Cauchy problem, periodic problem, Hille-Yosida operator, integrated semigroup, measure of non-compactness, fixed point, topological degree, multivalued map, condensing map.
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