

AN EXTENSION OF THE POINCARÉ-BIRKHOFF FIXED POINT THEOREM TO NONINVARIANT ANNULI

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Abstract. An extension of the Poincaré-Birkhoff fixed point theorem to noninvariant under area-preserving homeomorphism annuli is considered. Unlike the well-known W.-Y. Ding's theorem [7], the inner boundary component of an annulus is not assumed to be star-shaped, while the outer boundary component is star-shaped. The existence of at least two fixed points for area preserving homeomorphism satisfying some twist condition is proved.

Key Words and Phrases: Poincaré-Birkhoff fixed point theorem, noninvariant annulus, non-star-shaped boundary.

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