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POSITIVE SOLUTIONS FOR A FRACTIONAL BOUNDARY VALUE PROBLEM VIA A MIXED MONOTONE OPERATOR

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Abstract. In this paper, by using a mixed monotone operator method we study the existence and uniqueness of positive solutions to the following nonlinear fractional boundary value problem

$$\begin{cases} {}^{C}D^{\alpha}_{0^+}u(t) + f(t,u(t),(Hu)(t)) + g(t,u(t)) = 0, \quad 0 < t < 1, \\ u(0) = u''(0) = 0, \qquad\qquad u'(1) = \gamma u'(\eta), \end{cases}$$

where $2 < \alpha \leq 3$, $\gamma, \eta \in (0,1)$, ${}^{C}D^{\alpha}_{0^+}$ denotes de Caputo fractional derivative, $f : [0,1] \times [0,\infty) \times [0,\infty) \to [0,\infty)$ and $g : [0,1] \times [0,\infty) \to [0,\infty)$ are continuous functions and H is an operator (not necessarily linear) applying $\mathcal{C}[0,1]$ into itself. Moreover, in order to illustrate our results, we present some examples.

Key Words and Phrases: Fractional boundary value problem, positive solution, mixed monotone operator, fixed point.
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