DEGREE THEORY FOR DISCONTINUOUS OPERATORS

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Abstract. We introduce a new definition of topological degree for a meaningful class of operators which need not be continuous. Subsequently, we derive a number of fixed point theorems for such operators. As an application, we deduce a new existence result for first–order ODEs with discontinuous nonlinearities.

Key Words and Phrases: Degree theory, Leray–Schauder degree, Discontinuous differential equations.

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