

## ON AN EQUATION CHARACTERIZING MULTI-CUBIC MAPPINGS AND ITS STABILITY AND HYPERSTABILITY

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**Abstract.** In this paper, we introduce  $n$ -variables mappings which are cubic in each variable. We show that such mappings satisfy a functional equation. The main purpose is to extend the applications of a fixed point method to establish the Hyers-Ulam stability for the multi-cubic mappings. As a consequence, we prove that a multi-cubic functional equation can be hyperstable.

**Key Words and Phrases:** Banach space, Hyers-Ulam stability, multi-cubic mapping.

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