FIXED POINTS AND SETS OF MULTIVALUED CONTRACTIONS: AN ADVANCED SURVEY WITH SOME NEW RESULTS

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Dedicated to the memory of Professor Andrzej Granas (1929–2019)

Abstract. The existence of fixed points and, in particular, coupled fixed points is investigated for multivalued contractions in complete metric spaces. Multivalued coupled fractals are furthermore explored as coupled fixed points of certain induced operators in hyperspaces, i.e. as coupled compact subsets of the original spaces. The structure of fixed point sets is considered in terms of absolute retracts. We also formulate a continuation principle for multivalued contractions as a nonlinear alternative based on the topological essentiality. Two illustrative examples about coupled multivalued fractals are supplied.

Key Words and Phrases: Multivalued contractions, fixed points, coupled fixed points, coupled fractals, absolute retracts, continuation principle, essentiality.

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