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MODIFIED RELAXED CQ ALGORITHMS FOR SPLIT FEASIBILITY AND SPLIT EQUALITY PROBLEMS IN HILBERT SPACES

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Abstract. In this paper, we investigate the split feasibility problem (SFP) and the split equality problem (SEP) in Hilbert spaces. Motivated by the technique of relaxed projections, we respectively propose a modified relaxed CQ algorithm for the SFP and a modified relaxed alternating CQ algorithm for the SEP. Under standard assumptions, we show that the proposed algorithms converge weakly to a solution of the SFP and the SEP, respectively. Finally, we conduct some numerical experiments to demonstrate the advantage of our proposed algorithms.
Key Words and Phrases: Split feasibility problem, split equality problem, CQ algorithm, projection.
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