

ON THE ARONSZAJN PROPERTY FOR FRACTIONAL NEUTRAL EVOLUTION EQUATIONS WITH INFINITE DELAY ON HALF-LINE

NGUYEN NGOC TRONG*, LE XUAN TRUONG** AND NGUYEN THANH TUNG***

*Department of Primary Education, HCMC University of Education
E-mail: trongnn@hcmue.edu.vn

**Division of Computational Mathematics and Engineering, Institute for Computational Science
Ton Duc Thang University, Ho Chi Minh City, Vietnam
Faculty of Mathematics and Statistics, Ton Duc Thang University, Ho Chi Minh City
E-mail: lexuantruong@tdtu.edu.vn (Corresponding author)

***Department of Mathematics, HCMC University of Education
E-mail: thanhtungnt86@gmail.com

Abstract. We establish an R_δ structure theorem for the fixed point set of the Krasnosel'skii type operator on Fréchet space. Applying this result, a topological structure for the set of all mild solutions of fractional neutral evolution equations with infinite delay on half-line is investigated. We show that the solution set is an R_δ -set.

Key Words and Phrases: Topology structure, solution set, fractional differential equation, Krasnosel'skii type operator.

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