

ALGORITHMS FOR APPROXIMATING SOLUTIONS OF EQUILIBRIUM PROBLEMS AND FIXED POINTS OF NONEXPANSIVE-TYPE SEMIGROUP

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Abstract. In this paper, a Krasnoselskii-type and a Halpern-type algorithms for approximating a fixed point of a totally quasi- ϕ -asymptotically nonexpansive multi-valued semigroup and a solution of a generalized mixed equilibrium problem is studied. Strong convergence of the sequences generated by these algorithms is proved in real Banach spaces. Finally, the theorems proved are significant improvement on several important recent results.

Key Words and Phrases: Halpern-type algorithm, Krasnoselskii-type algorithm, generalized mixed equilibrium problems, totally quasi- ϕ -asymptotically nonexpansive multi-valued maps, equally continuous maps.

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