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AN ULTRA-PRODUCT METHOD VIA LEFT REVERSIBLE SEMIGROUPS TO STUDY BRUCK'S GENERALIZED CONJECTURE

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Abstract. It has been asked by Lau several times whether a Banach space with weak fixed point property has weak fixed point property for left reversible semigroups. This problem is known as Bruck generalized conjecture (BGC). The aim of this note is to propose a new approach to tackle the BGC. Our approach uses the order structure of the semigroup for the first time in literature to construct an ultra-product structure. Then, we use this ultra-product structure to give an affirmative answer to BGC for the case of nearly uniformly convex (NUC) Banach spaces. One should note that alternatives proofs are available in the case of NUC Banach spaces, but what we hope for is that the originality of our method could pave the way for studying the BGC in its utmost generality.
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