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NONLOCAL SOLUTIONS AND CONTROLLABILITY OF SCHRÖDINGER EVOLUTION EQUATION

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Abstract. The paper deals with semilinear evolution equations in complex Hilbert spaces. Nonlocal associated Cauchy problems are studied and the existence and uniqueness of classical solutions is proved. The controllability is investigated too and the topological structure of the controllable set discussed. The results are applied to nonlinear Schrödinger evolution equations with time dependent potential. Several examples of nonlocal conditions are proposed. The evolution system associated to the linear part is not compact and the theory developed in Okazawa-Yoshii [21] for its study is used. The proofs involve the Schauder-Tychonoff fixed point theorem and no strong compactness is assumed on the nonlinear part.

Key Words and Phrases: Schrödinger equation, potential with singularities, existence and uniqueness of C^1-solutions, nonlocal conditions, controllability, fixed point theorems.
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