

## SYSTEMS OF UNRELATED GENERALIZED MIXED EQUILIBRIUM PROBLEMS AND UNRELATED HIERARCHICAL FIXED POINT PROBLEMS IN HILBERT SPACE

K.R. KAZMI<sup>\*,\*\*</sup>, SALEEM YOUSUF<sup>\*\*</sup> AND REHAN ALI<sup>\*\*\*</sup>

<sup>\*</sup>Department of Mathematics, Faculty of Science & Arts - Rabigh  
King Abdulaziz University, P.O. Box 344, Rabigh 21911, Kingdom of Saudi Arabia  
E-mail: [krkazmi@gmail.com](mailto:krkazmi@gmail.com) (Corresponding author)

<sup>\*\*</sup>Department of Mathematics, Aligarh Muslim University, Aligarh 202002, India  
E-mail: [saleemamu12@gmail.com](mailto:saleemamu12@gmail.com)

<sup>\*\*\*</sup>Department of Mathematics, Jamia Millia Islamia, New Delhi 110025, India  
E-mail: [rehan08amu@gmail.com](mailto:rehan08amu@gmail.com)

**Abstract.** In this paper, we investigate a hybrid extra-gradient iterative method to approximate the common solution of a system of unrelated generalized mixed equilibrium problems for monotone and Lipschitz continuous mappings and system of unrelated hierarchical fixed point problems for nonexpansive mappings in Hilbert space. We prove a strong convergence theorem for the sequences generated by the proposed iterative algorithm. Further, we give some consequences and applications of our main result. Finally, we discuss a numerical example to demonstrate the applicability of the iterative algorithm.

**Key Words and Phrases:** System of unrelated generalized mixed equilibrium problems, system of unrelated hierarchical fixed point problems, monotone mapping, Lipschitz continuous mapping, nonexpansive mapping, hybrid extra-gradient iterative method.

**2010 Mathematics Subject Classification:** 47H05, 47H09, 47J25, 47H10.

**Acknowledgments.** The authors are very thankful to the anonymous referees for their critical comments and helpful suggestions which led to substantial improvements in the original version of the manuscript.

### REFERENCES

- [1] H. Brezis, *Operateurs maximaux monotones et semi-groupes de contractions dans les espaces de Hilbert*, Mathematical Studies, Amsterdam: North-Holand, **5**(1973), 759-775.
- [2] A. Cabot, *Proximal point algorithm controlled by a slowly vanishing term: application to hierarchical minimization*, *SIAM J. Optim.*, **15**(2005), 555-572.
- [3] Y. Censor, A. Gibali, S. Reich, S. Sabach, *Common solutions to variational inequality*, *Set-Valued Var. Anal.*, **20**(2012), 229-247.
- [4] B. Djafari-Rouhani, M. Farid, K.R. Kazmi, *Common solution to generalized mixed equilibrium problem and fixed point problem for a nonexpansive semigroup in Hilbert space*, *J. Korean Math. Soc.*, **53**(1)(2016), 89-114.

- [5] B. Djafari-Rouhani, K.R. Kazmi, S.H. Rizvi, *A hybrid-extragradient-convex approximation method for a system of unrelated mixed equilibrium problems*, Trans. Math. Pogram. Appl., **1**(8)(2013), 82-95.
- [6] H. Iiduka, W. Takahashi, M. Toyoda, *Approximation of solutions of variational inequalities for monotone mappings*, Panamer. Math. J., **14**(2004), 49-61.
- [7] K.R. Kazmi, R. Ali, M. Furkan, *Krasnoselski-Mann type iterative method for hierarchical fixed point problem and split mixed equilibrium problem*, Numerical Algorithms, **77**(1)(2018), 289-308.
- [8] K.R. Kazmi, R. Ali, M. Furkan, *Hybrid iterative method for split monotone variational inclusion problem and hierarchical fixed point problem for a finite family of nonexpansive mappings*, Numerical Algorithms, **79**(2)(2018), 499-527.
- [9] G.M. Korpelevich, *The extragradient method for finding saddle points and other problems*, Matecon, **12**(1976), 747-756.
- [10] Z.Q. Luo, J.S. Pang, D. Ralph, *Mathematical Programs with Equilibrium Constraints*, Cambridge, Cambridge University Press, 1996.
- [11] A. Moudafi, *Krasnoselski-Mann iteration for hierarchical fixed-point problems*, Inverse Probl., **23**(2007), 1635-1640.
- [12] A. Moudafi, P.E. Mainge, *Towards viscosity approximations of hierarchical fixed-point problems*, Fixed Point Theory Appl., **2006**(2006), Art. ID 95453, 10 pages.
- [13] A. Moudafi, P.E. Mainge, *Strong convergence of an iterative method for hierarchical fixed-point problems*, Pacific J. Optim., **3**(2007), 529-538.
- [14] N. Nadezhkina, W. Takahashi, *Strong convergence theorem by a hybrid method for nonexpansive mappings and Lipschitz continuous monotone mappings*, SIAM J. Optim., **16**(40)(2006), 1230-1241.
- [15] K. Nakajo, K., W. Takahashi, *Strong convergence theorems for nonexpansive mappings and nonexpansive semigroups*, J. Math. Anal. Appl., **279**(2003), 372-379.
- [16] M. Sofonea, A. Matei, *Mathematical Models in Contact Mechanics*, London Mathematical Society Lecture Note Series 398, Cambridge University Press, 2012.
- [17] M. Sofonea, S. Migorski, *Variational-Hemivariational Inequalities with Applications*, CRC Press, Taylor & Francis LLC, 2018.

*Received: June 7, 2018 ; Accepted: October 31, 2019.*

