VISCOSITY METHOD WITH A ϕ-CONTRACTION MAPPING FOR HIERARCHICAL VARIATIONAL INEQUALITIES ON HADAMARD MANIFOLDS

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Abstract. In this paper, we propose the viscosity method for solving variational inequality problems defined over a set of fixed points of a nonexpansive mapping and involving a ϕ-contraction mapping and another nonexpansive mapping in the setting of Hadamard manifolds. Several special cases of such a variational inequality problem are also considered. The convergence analysis of the proposed method is studied. We illustrate proposed algorithm and convergence result by a numerical example. The algorithms and convergence results of this paper extend and improve several known algorithms and results from linear structure to Hadamard manifolds.

Key Words and Phrases: Viscosity method, ϕ-contraction mappings, hierarchical variational inequality problem, Moreau-Yosida regularization, hierarchical minimization problem, Hadamard manifolds, monotone vector fields, geodesic convexity, nonexpansive mappings.

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