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EXISTENCE OF THREE WEAK SOLUTIONS FOR KIRCHHOFF-TYPE PROBLEMS WITH VARIABLE EXPONENT AND NONHOMOGENEOUS NEUMANN CONDITIONS

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Abstract. We study the existence of at least three weak solutions for a class of differential equations with $p(x)$-Kirchhoff-type and subject to perturbations of nonhomogeneous Neumann conditions. Our technical approach is based on variational methods. Some applications and examples illustrate the obtained results.

Key Words and Phrases: Variable exponent Sobolev spaces, $p(x)$-Kirchhoff-type problems, three weak solutions, variational methods.
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