

PACKING, w_n^* – SEPARATION AND NORMAL STRUCTURE IN BANACH SPACES

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Abstract. Let X and X^* be a Banach space and its dual, and let $B(X)$ and $S(X)$ be the unit ball and unit sphere of X respectively. In this paper, we introduce a new parameter of w_n^* – Separation, $w_n^*(X^*)$, in X^* and study the relation between this parameter and normal structure in X , and the relation between packing constant $P(\alpha, X)$ introduced by Kottman and normal structure that implies the existence of fixed point for non-expansive mappings. Some new results about fixed points of non-expansive mapping are obtained.

Key Words and Phrases: Fixed points, normal structure, packing, ultra-product, weak normal structure.

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