

SEVERAL FIXED POINT THEOREMS ON PARTIALLY ORDERED BANACH SPACES AND THEIR APPLICATIONS TO INTEGRAL EQUATIONS

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Abstract. In this paper, we prove several fixed point theorems on partially ordered Banach spaces, in which the considered mappings satisfy some order monotone conditions. In addition, if the considered mappings satisfy some continuity conditions, then some iterated schemes can be constructed to approximate the fixed points. Then we apply these theorems to prove an existence theorem of solutions to some Hammerstein integral equations.

Key Words and Phrases: Partially ordered Banach space, regular partially ordered Banach space, fixed point, C-continuity of set-valued mappings, Hammerstein integral equations.

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