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EXISTENCE RESULTS FOR A QUADRATIC INTEGRAL EQUATION OF FRACTIONAL ORDER BY A CERTAIN FUNCTION

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Abstract. The fractional integration of a function $f(t)$ by a function ϕ and some of its properties is presented in [23], [30] and [21]. As an application for this fractional integration we present some existence results for at least one continuous solution for a nonlinear quadratic functional integral equation of fractional (arbitrary) order. Also, some examples and remarks are illustrated. Finally, we prove the existence of maximal and minimal solutions for that equations.
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References

[1] I.K. Argyros, *Quadratic equations and applications to Chandrasekhar's and related equations*, Bull. Austral. Math. Soc., **32**(1985), 275-292.
[2] I.K. Argyros, *On a class of quadratic integral equations with perturbations*, Funct. Approx., **20**(1992), 51-63.
[3] J. Banaś, J. Caballero, J. Rocha, K. Sadarangani, *Monotonic solutions of a class of quadratic integral equations of Volterra type*, Comput. Math. Appl., **49**(2005), 943-952.
[4] J. Banaś, M. Lecko, W.G. El-Sayed, *Existence theorems of some quadratic integral equation*, J. Math. Anal. Appl., **227**(1998), 276-285.
[5] J. Banaś, A. Martinon, *Monotonic solutions of a quadratic integral equation of Volterra type*, Comput. Math. Appl., **47**(2004), 271-279.
[6] J. Banaś, J. Rocha Martin, K. Sadarangani, *On the solution of a quadratic integral equation of Hammerstein type*, Math. Computer Model., **43**(2006), 97-104.
[7] J. Banaś, B. Rzepka, *Monotonic solutions of a quadratic integral equations of fractional order*, J. Math. Anal. Appl., **332**(2007), 1370-11378.
[8] J. Banaś, B. Rzepka, *Nondecreasing solutions of a quadratic singular Volterra integral equation*, Math. Comput. Modelling, **49**(2009), 488-496.

[9] M. Cichon, M.A. Metwali, *On quadratic integral equations in Orlicz spaces*, J. Math. Anal. Appl., **387**(2012), no. 1, 419-432.
[10] R.F. Curtain, A.J. Pritchard, *Functional Analysis in Modern Applied Mathematics*, Academic Press, 1977.
[11] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, Berlin, 1985.
[12] M.M. El-Borai, W.G. El-Sayed, M.I. Abbas, *Monotonic solutions of a class of quadratic singular integral equations of Volterra type*, Int. J. Contemp. Math. Sci., **2**(2007), no. 2, 89-102.
[13] A.M.A. El-Sayed, F.M. Gaafar, H.H.G. Hashem, *On the maximal and minimal solutions of arbitrary-orders nonlinear functional integral and differenbtial equations*, Math. Sci, Res. J., **8(11)**(2004), 336-348.
[14] A.M.A. El-Sayed, H.H.G. Hashem, *Carathèodory type theorem for a nonlinear quadratic integral equation*, Math. Sci. Res. J., **12**(2008), no. 4, 71-95.
[15] A.M.A. El-Sayed, H.H.G. Hashem, *Monotonic positive solution of nonlinear quadratic Hammerstein and Urysohn functional integral equations*, Commentationes Math., **48**(2008), no. 2, 199-207.
[16] A.M.A. El-Sayed, H.H.G. Hashem, *Monotonic solutions of functional integral and differential equations of fractional order*, Electronic J. Qualitative Th. Differential Eq., **7**(2009), 1-8.
[17] A.M.A. El-Sayed, H.H.G. Hashem, *Monotonic positive solution of a nonlinear quadratic functional integral equation*, Appl. Math. and Comput., **216**(2010), 2576-2580.
[18] A.M.A. El-Sayed, H.H.G. Hashem, *Existence results for nonlinear quadratic functional integral equations of fractional order*, Miskolc Mathematical Notes, **14**(2013), no. 1, 79-87.
[19] A.M.A. El-Sayed, H.H.G. Hashem, E.A.A. Ziada, *Picard and Adomian Methods for quadratic integral equation*, Comput. Appl. Math., **29**(2010), no. 3, 447-463.
[20] W.G. El-Sayed, B. Rzepka, *Nondecreasing solutions of a quadratic integral equation of Urysohn type*, Comput. Math. Appl., **51**(2006), 1065-1074.
[21] H.H.G. Hashem, *On the solution of a generalized fractional order integral equation and some applications*, J. Fractional Calculus Appl., **6**(2015), no. 1, 120-130.
[22] H.H.G. Hashem, M.S. Zaki, *Carathèodory theorem for quadratic integral equations of Erdyéli-Kober type*, J. Fractional Calculus Appl., **4**(2013), no. 5, 1-8.
[23] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Elsevier, North-Holland, 2006.
[24] V. Lakshmikantham, S. Leela, *Differential and Integral Inequalities*, vol. 1, Academic Press, New York-London, 1969.
[25] G. Pagnini, *Erdélyi-Kober fractional diffusion*, Fract. Calculus and Appl. Anal., **15**(2012), no. 1, 117-127.
[26] I. Podlubny, *Fractional Differential equations*, San Diego – New York – London, 1999.
[27] M.R. Rao, *Ordinary Differential Equations*, East-West Press, 1980.
[28] B. Ross, K.S. Miller, *An Introduction to Fractional Calculus and Fractional Differential Equations*, John Wiley, New York, 1993.
[29] H.A.H. Salem, *On the quadratic integral equations and their applications*, Computers and Mathematics with Applications, **62**(2011), 2931-2943.
[30] S.G. Samko, A.A. Kilbas, O. Marichev, *Integrals and Derivatives of Fractional Orders and Some of their Applications*, Nauka i Teknika, Minsk, 1987.

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