A STRONG CONVERGENCE THEOREM FOR FINITE FAMILIES OF BREGMAN QUASI-NONEXPANSIVE AND MONOTONE MAPPINGS IN BANACH SPACES

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Abstract. We introduce a new iterative scheme and prove a strong convergence theorem for it. This iterative scheme finds a common point in the set of fixed points of a finite family of Bregman quasi-nonexpansive mappings and the common solution set of the variational inequality problem for continuous monotone mappings.
Key Words and Phrases: Bregman quasi-nonexpansive mappings, strong convergence, continuous monotone mappings, fixed point.
2010 Mathematics Subject Classification: 47H09, 47H10, 47J25, 47H05, 4705.

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